Homework 10/11

19.

For the given hypotheses, the test statistic $t = \frac{115.7 - 129.3 + 10}{\sqrt{\frac{5.03^2}{6} + \frac{5.38^2}{6}}} = \frac{-3.6}{3.007} = -1.20$, and the d.f. is

$\nu = \frac{(4.2168 + 4.8241)^2}{\frac{(4.2168)^2}{5} + \frac{(4.8241)^2}{5}} = 9.96$, so use d.f. = 9. We will reject H_0 if $t \le -t_{.01,9} = -2.764;$ since

–1.20 > -2.764, we don't reject H_0.

20.

We want a 95% confidence interval for $\mu_1 - \mu_2$. $t_{.025,9} = 2.262$, so the interval is $-13.6 \pm 2.262(3.007) = (-20.40, -6.80)$. Because the interval is so wide, it does not appear that precise information is available.

28.

We will test the hypotheses: $H_0 : \mu_1 - \mu_2 = 10$ vs. $H_a : \mu_1 - \mu_2 > 10$. The test statistic is

$t = \frac{(\bar{x} - \bar{y}) - 10}{\sqrt{\left(\frac{2.75^2}{10} + \frac{4.44^2}{5}\right)}} = \frac{4.5}{2.17} = 2.08$ The degrees of freedom

$\nu = \frac{\left(\frac{2.75^2}{10} + \frac{4.44^2}{5}\right)^2}{\frac{\left(\frac{2.75^2}{10}\right)^2}{9} + \frac{\left(\frac{4.44^2}{5}\right)^2}{4}} = \frac{22.08}{3.95} = 5.59 \approx 5$, and the p-value from table A.8 is approx .045, which is < .10 so we reject H_0 and conclude that the true average lean angle for older females is more than 10 degrees smaller than that of younger females.

34.

a. Following the usual format for most confidence intervals: *statistic ±(critical value)(standard error),* a pooled variance confidence interval for the difference between two means is $(\bar{x} - \bar{y}) \pm t_{\alpha/2, m+n-2} \cdot s_p \sqrt{\frac{1}{m} + \frac{1}{n}}$.

b. The sample means and standard deviations of the two samples are $\bar{x} = 13.90$, $s_1 = 1.225$, $\bar{y} = 12.20$, $s_2 = 1.010$. The pooled variance estimate is $s_p^2 =$

$\left(\frac{m-1}{m+n-2}\right)s_1^2 + \left(\frac{n-1}{m+n-2}\right)s_2^2 = \left(\frac{4-1}{4+4-2}\right)(1.225)^2 + \left(\frac{4-1}{4+4-2}\right)(1.010)^2 = 1.260$, so $s_p = 1.1227$. With df = m+n-1 = 6 for this interval, $t_{.025,6} = 2.447$ and the desired interval is $(13.90-12.20) \pm (2.447)(1.1227)\sqrt{\frac{1}{4}+\frac{1}{4}} = 1.7 \pm 1.943 = (-.24, 3.64)$. This interval contains 0, so it does not support the conclusion that the two population means are different.

c. Using the two-sample t interval discussed earlier, we use the CI as follows: First, we need to calculate the degrees of freedom. $\nu = \dfrac{\left(\frac{1.225^2}{4}+\frac{1.01^2}{4}\right)^2}{\frac{\left(\frac{1.225^2}{4}\right)^2}{3}+\frac{\left(\frac{1.01^2}{4}\right)^2}{3}} = \dfrac{.3971}{.0686} = 5.78 \downarrow 5$ so $t_{.025,5} = 2.571$. Then the interval is $(13.9-12.2) \pm 2.571\sqrt{\frac{1.225^2}{4}+\frac{1.01^2}{4}} = 1.70 \pm 2.571(.7938) = (-.34, 3.74)$. This interval is slightly wider, but it still supports the same conclusion.

40.

From the data, n = 10, $\bar{d}$ = 105.7, s_d = 103.845.

a.

Let μ_d = true mean difference in TBBMC, postweaning minus lactation. We wish to test the hypotheses H_0: $\mu_d \le 25$ v. H_a: $\mu_d > 25$. The test statistic is $t = \dfrac{105.7-25}{103.845/\sqrt{10}}$ = 2.46; at 9df, the corresponding P-value is around .018. Hence, at the 5% significance level, we reject H_0 and conclude that true average TBBMC during postweaning does exceed the average during lactation by more than 25 grams.

b. A 95% upper confidence bound for μ_d = $\bar{d} + t_{.05,9}s_d/\sqrt{n}$ = 105.7 + 1.833(103.845)/$\sqrt{10}$ = 165.89 grams.

c. No. If we pretend the two samples are independent, the new standard error is is roughly 235, far greater than 103.845/$\sqrt{10}$. In turn, the resulting t statistic is just t = 0.45, with estimated df = 17 and P-value = .329 (all using a computer).

48.

a. H_0 will be rejected if $|z| \ge 1.96$. With $\hat{p}_1 = \dfrac{63}{300} = .2100$, and $\hat{p}_2 = \dfrac{75}{180} = .4167$,

$$\hat{p} = \frac{63+75}{300+180} = .2875,\ z = \frac{.2100-.4167}{\sqrt{(.2875)(.7125)\left(\frac{1}{300}+\frac{1}{180}\right)}} = \frac{-.2067}{.0427} = -4.84.$$ Since $-4.84 \le -1.96$, H_0 is rejected.

b.

$\overline{p} = .275$ and $\sigma = .0432$, so power = $1 - \left[\Phi\left(\frac{[(1.96)(.0421) + .2]}{.0432} \right) - \Phi\left(\frac{[-(1.96)(.0421) + .2]}{.0432} \right) \right] =$
$1 - [\Phi(6.54) - \Phi(2.72)] = .9967$.

50.

Let $\alpha = .05$. A 95% confidence interval is $(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\left(\frac{\hat{p}_1\hat{q}_1}{m} + \frac{\hat{p}_2\hat{q}_2}{n}\right)}$

$= \left(\frac{224}{395} - \frac{126}{266}\right) \pm 1.96\sqrt{\left(\frac{\left(\frac{224}{395}\right)\left(\frac{171}{395}\right)}{395} + \frac{\left(\frac{126}{266}\right)\left(\frac{140}{266}\right)}{266}\right)} = .0934 \pm .0774 = (.0160, .1708)$.

57.

a. From Table A.9, column 5, row 8, $F_{.01,5,8} = 3.69$.

b. From column 8, row 5, $F_{.01,8,5} = 4.82$.

c. $F_{.95,5,8} = \frac{1}{F_{.05,8,5}} = .207$.

d. $F_{.95,8,5} = \frac{1}{F_{.05,5,8}} = .271$

e. $F_{.01,10,12} = 4.30$

f. $F_{.99,10,12} = \frac{1}{F_{.01,12,10}} = \frac{1}{4.71} = .212$.

g. $F_{.05,6,4} = 6.16$, so $P(F \le 6.16) = .95$.

h. Since $F_{.99,10,5} = \frac{1}{5.64} = .177$, $P(.177 \le F \le 4.74) = P(F \le 4.74) - P(F \le .177) = .95 - .01 = .94$.

62.

For the hypotheses $H_0: \sigma_1 = \sigma_2$ versus $H_a: \sigma_1 \neq \sigma_2$, we find a test statistic of $f = 1.22$. At df = $(47,44) \approx (40,40)$, $1.22 < 1.51$ indicates the P-value is greater than $2(.10) = .20$. Hence, H_0 is not rejected. The data does not suggest a significant difference in the two population variances.