

## Practice Final

1. In a health science study, 15 healthy men were randomly selected and their systolic blood pressure level will be measured. Their systolic blood pressure levels are assumed to follow a normal distribution  $N(\mu, \sigma^2)$ . The 15 sample yielded a sample mean 127 and sample standard deviation 15.
  - (a) Construct a 90 % confidence interval for  $\mu$ .
  - (b) Test hypothesis  $H_0 : \mu = 120$  versus  $H_a : \mu > 120$  with  $\alpha = 0.05$ .
  - (c) Suppose  $\sigma = 2$ . Find  $n$  necessary to estimate  $\mu$  within 0.5 with probability 0.90.
  
2. An experiment was conducted to compare the weight of women in 20's living in Wisconsin and Minnesota. 40 Wisconsin women were examined and found to have sample mean 140 lb while 38 Minnesotan women yielded the sample mean 149 lb. Assume the weights (in pound) follow normal distributions  $N(\mu_1, \sigma_1^2)$  (WI) and  $N(\mu_2, \sigma_2^2)$  (MN), respectively.
  - (a) Assume that  $\sigma_1 = 5$  and  $\sigma_2 = 7$ . Test  $H_0 : \mu_1 = \mu_2$  versus  $H_a : \mu_1 > \mu_2$  with  $\alpha = 0.05$ .
  - (b) Let  $m$  and  $n$  be the sample sizes for WI and MN, respectively. Meeting MN women is harder so the experimenter wants to have  $n = 2m$ . Find  $m$  and  $n$  such that a level 0.01 test has type II error 0.1 at  $\mu_1 - \mu_2 = 3$ .
  
3. In a technical trouble shooting call center, 10 randomly selected phone calls were monitored. Let  $p$  be the true proportion of satisfied customers, and  $X$  be the number of satisfied customers. It was found that  $X = 4$ .
  - (a) Let  $L(p)$  be the likelihood function. Find  $\frac{d}{dp} \log L(p)$ .
  - (b) Consider hypothesis  $H_0 : p = 0.5$  versus  $H_a : p < 0.5$ . For a test procedure with rejection region  $X \leq 2$ , find the probability of Type II error.

4. Suppose the number of rare inheritable disease patients (in an year) follows a Poisson distribution

$$f(x; \theta) = \frac{e^{-\theta} \theta^x}{x!}, x = 0, 1, 2, \dots$$

Suppose the number of participants were counted for 70 years, and the sample average for 70 years was 2.2, and the standard deviation was 1.7.

- (a) Find the MLE of  $\theta$ .
- (b) Find the p-value for testing  $H_0 : \theta = 2$  versus  $H_a : \theta > 2$ .