1.3 Definitions and Examples

Definition: A random variable, usually denoted by $X$, $Y$, $Z$,..., is a rule that assigns a number to each outcome of an experiment. (Examples: $X$ = mass, pulse rate, gender)

Definition: Statistics is a collection of formal computational techniques that are designed to test and derive a (reject or “accept”) conclusion about a null hypothesis for a random variable defined on a population, based on experimental data taken from a random sample.

- Example: Blood sample taken from a patient for medical testing purposes, and results compared with ideal reference values, to see if differences are significant.

- Example: “Goldilocks Principle”

\[ \text{POPULATION} = \text{swimming pool} \]

Random Variable

$X$ = Water Temperature (°F)

(Informal) Null Hypothesis

$H_0$: “(The mean of) $X$ is okay for swimming.” (e.g., $\mu = 80^\circ\text{F}$)

(Informal) Experiment

Select a random sample by sticking in foot and swishing water around.

(Informal) Analysis

Determine if the difference between the observed temperature and expected temperature under $H_0$ is significant.

Conclusion

If not, then accept $H_0$… Jump in!
If so, then reject $H_0$… Go jogging instead.

The following example illustrates the general approach used in formal hypothesis testing.

- Example: United States criminal justice system

\[ \text{Null Hypothesis } H_0: \text{ “Defendant is innocent.”} \]

The burden of proof is on the prosecution to collect enough empirical evidence to try to reject this hypothesis, “beyond a reasonable doubt” (i.e., at some significance level).

Jodi Arias
CONVICTED
$H_0$ rejected
May 8, 2013

Casey Anthony
ACQUITTED
$H_0$ “accepted”
July 5, 2011
Example: Pharmaceutical Application

**Phase III Randomized Clinical Trial (RCT)**

- Used to compare “drug vs. placebo,” “new treatment vs. standard treatment,” etc., via randomization (to eliminate bias) of participants to either a treatment arm or control arm. Moreover, randomization is often “blind” (i.e., “masked”), and implemented by computer, especially in multicenter collaborative studies. Increasing use of the Internet!
- Standard procedure used by FDA to approve pharmaceuticals and other medical treatments for national consumer population.

**POPULATION**

**Random Variable** “\( X = \) cholesterol level (mg/dL)”

**Drug** \( \mu_1 \)  
**Placebo** \( \mu_2 \)

**RANDOM SAMPLES**

Size \( n_1 \)  
Size \( n_2 \)

\( \bar{x}_1 = 225 \)  
\( \bar{x}_2 = 240 \)

Null Hypothesis

\( H_0: \) There is no difference in population mean cholesterol levels between the two groups, i.e., 
\[ \mu_1 - \mu_2 = 0. \]

Is the mean difference statistically significant, (e.g., at the \( \alpha = .05 \) level)?
- If so, then reject \( H_0 \).
- If not, then “accept” \( H_0 \). There is not enough evidence of a genuine treatment difference.

More study needed?

\( \bar{x}_1 - \bar{x}_2 = -15 \)