5. Sampling Distributions and the Central Limit Theorem

5.1 Motivation

**POPULATION = U.S. Adult Males**
Random Variable $X = \text{Height (inches)}$

**Random Samples (all of size $n$)**

**Population Distribution of $X$**
- $\mu_X = 70$
- $\sigma_X = 4$
- **TYPICAL** $x \approx 70$
- **RARE** – short outlier $x << 70$
- **RARE** – tall outlier $x >> 70$

**Sampling Distribution of $\bar{X}$**
- $\mu_{\bar{X}} = 70$
- $\sigma_{\bar{X}} = \frac{4}{\sqrt{n}}$
- **EXTREMELY TYPICAL** $\bar{x} \approx 70$
- **EXTREMELY RARE** – mostly short outliers $\bar{x} << 70$
- **EXTREMELY RARE** – mostly tall outliers $\bar{x} >> 70$

**Random Sample**
- mostly near the population mean
- with a few short and tall outliers

**Random Sample**
- mostly short outliers
- with a few short and tall outliers

**Random Sample**
- mostly tall outliers
- with a few short and tall outliers