1. Three masses of 5 grams, 3 grams, and 2 grams are centered on the 2 inch, 8 inch, and 11 inch marks of a ruler, respectively. The “balance point” of this system is located at

(a) 5.2 inches
(b) 5.6 inches
(c) 6.0 inches
(d) 6.4 inches
(e) None of the above

2. The continuous random variable $X$ with density function

$$f(x) = \begin{cases} 
\frac{2x}{15}, & 0 \leq x < 3 \\
\frac{2}{5}, & 3 \leq x \leq 4 \\
0, & \text{otherwise}
\end{cases}$$

and 0 otherwise, has mean equal to

(a) 1.3
(b) 2.4
(c) 2.6
(d) 2.8
(e) None of the above

3. Suppose it is known that 40% of a certain population suffers from either one or both of two medical conditions. In particular, it is determined that a total of 20% have the first condition, and a total of 25% have the second. From this information, we may conclude that

(a) there is statistical independence between the two conditions in this population
(b) there is statistical dependence between the two conditions in this population
(c) the two conditions are neither statistically independent nor dependent in this population
(d) Not enough information is given to form a conclusion about statistical independence.
(e) None of the above

4. In a prospective cohort study investigating the association between a certain disease and exposure to a potential risk factor, it is reported that the odds ratio is 6.0, and the relative risk is 2.0. What is the probability of developing the disease, given exposure?

(a) 80%
(b) 70%
(c) 60%
(d) Not enough information is given to calculate this probability.
(e) None of the above
5. The hazard rate of a certain population is given by \( h(t) = 0.09 \sqrt{t} \), for \( t \geq 0 \) (in years). From among those individuals who have already survived beyond 4 years, the probability of surviving beyond 9 years is

(a) 32%
(b) 41%
(c) 57%
(d) 68%
(e) None of the above

6. All of the individuals in a certain large experiment undergo a diet and/or exercise regimen in order to lose weight. In particular, a total of 80% of them diet, a total of 70% of them exercise, and 50% do both. It is eventually found that significant weight loss occurs in 10% of those who only dieted, 35% of those who only exercised, and 90% in those who did both. What overall percentage of individuals experienced significant weight loss in this experiment?

(a) 65%
(b) 60%
(c) 55%
(d) 50%
(e) None of the above

7. Which of the following statements is true?

(a) If an experimental result is statistically significant at the 5% level, it is also statistically significant at the 1% level.
(b) The \( p \)-value of a one-sided test is always half that of the corresponding two-sided test.
(c) In a test of a null hypothesis, the probability of correctly accepting it when true is called the power of the test.
(d) Two disjoint events (with nonzero probabilities) cannot be statistically independent.
(e) None of the above

8. A scatterplot of \( n = 16 \) data points has response variance \( s_y^2 = 5.0 \), and correlation coefficient \( r = 0.8 \). The least squares regression line will have its residual sum of squares, \( SS_{Err} \), equal to

(a) 27.0
(b) 48.0
(c) 60.0
(d) Not enough information is given to calculate \( SS_{Err} \).
(e) None of the above
9. In a certain normally-distributed population of widgets, it is known that 97% weigh less than 533 pounds, and 96% weigh less than 520 pounds. What percentage weigh less than 389 pounds?

(a) 67%
(b) 58.5%
(c) 50%
(d) 44%
(e) None of the above

10. In testing a certain "Y-pipe" plumbing system, valves A and B on two vertical pipes, both leading to a common, central spigot that flows into a tank, are periodically opened and closed at random. Valve A is opened with probability 0.4, and independently, valve B is opened with probability 0.3. With what probability is water flowing into the tank?

(a) .70
(b) .58
(c) .35
(d) .12
(e) None of the above
Solutions

1. (b)

The balance point of the values 2, 8, and 11 is their weighted average, using “weights” 5g/10g = 0.5, 3g/10g = 0.3, and 2g/10g = 0.2, respectively. Hence, \((2)(0.5) + (8)(0.3) + (11)(0.2) = 5.6\) inches.

2. (c)

By definition, 
\[
\mu = \int_{-\infty}^{\infty} x f(x) \, dx = \int_{0}^{3} x \left(\frac{2x}{15}\right) \, dx + \int_{3}^{4} x \left(\frac{2}{5}\right) \, dx = \frac{2}{15} \left[\frac{x^3}{3}\right]_{0}^{3} + \frac{2}{5} \left[\frac{x^2}{2}\right]_{3}^{4} = \frac{6}{5} + \frac{7}{5} = \frac{13}{5} = 2.6.
\]

3. (a)

The problem states that, for events \(A\) and \(B\), \(P(A \cup B) = 0.4\), \(P(A) = 0.2\), \(P(B) = 0.25\). Because \(P(A \cup B) = P(A) + P(B) - P(A \cap B)\), it follows that \(0.4 = 0.2 + 0.25 - P(A \cap B)\), so that \(P(A \cap B) = 0.05\). But \(P(A)P(B) = (0.2)(0.25) = 0.05\) also. Hence, we have the relation that \(P(A \cap B) = P(A)P(B)\), which indicates statistical independence.

4. (a)

Recall that relative risk \(RR\) is a ratio of probabilities, odds ratio \(OR\) is a ratio of odds. Therefore, with notation \(p = P(D+ | E+)\) and \(q = P(D+ | E-)\), we have \(RR = \frac{p}{q}\) and 
\[
OR = \frac{\frac{p}{1-p}}{\frac{q}{1-q}} = \frac{p}{1-p} = \frac{p(1-q)}{q(1-p)}.
\]

The problem gives \(RR = \frac{p}{q} = 2\) (i.e., \(p = 2q\)) and \(OR = \frac{p(1-q)}{q(1-p)} = 6\).

Solving these two equations simultaneously yields \(q = 0.4\), and \(p = 0.8\), or 80%.

5. (a)

If \(h(t) = 0.09 t^{1/2}\), then the survival function is \(S(t) = e^{-\int_0^t h(x) \, dx} = e^{-\int_0^t 0.09 x^{1/2} \, dx} = e^{-0.06 x^{3/2}}\). (Note this is a Weibull model.) Thus, the conditional probability \(P(T > 9 | T > 4) = \frac{P(T > 9)}{P(T > 4)} = \frac{S(9)}{S(4)} = \frac{e^{-0.06(9^{3/2})}}{e^{-0.06(4^{3/2})}} = \frac{e^{-0.06(27)}}{e^{-0.06(16)}} = e^{-0.06(1)} = e^{-1.14} = 0.32\), or 32%.
6. (c)

With events $D = \text{diet}$, $E = \text{exercise}$, the problem states that $P(D) = 0.8$, $P(E) = 0.7$, and $P(D \cap E) = 0.5$. The next statement in the problem involves the three disjoint events $B_1 = D \text{ only}$, $B_2 = D \cap E$, and $B_3 = E \text{ only}$. So, $P(B_1) = 0.8 - 0.5 = 0.3$, $P(B_2) = 0.5$, and $P(B_3) = 0.7 - 0.5 = 0.2$, whose probabilities sum to 1. Therefore, these three disjoint events are also exhaustive, i.e., their union is the entire population. (Note that this is consistent with the very first statement in the problem.) We are also told that, for event $A = \text{significant weight loss}$, $P(A \mid B_1) = 0.1$, $P(A \mid B_2) = 0.9$, and $P(A \mid B_3) = 0.35$. Hence, from the Law of Total Probability, we obtain $P(A) = P(A \mid B_1)P(B_1) + P(A \mid B_2)P(B_2) + P(A \mid B_3)P(B_3) = (0.1)(0.3) + (0.9)(0.5) + (0.35)(0.2) = 0.55$, or 55%.

7. (d)

(a) is FALSE: For example, a $p$-value of .02 is significant at (i.e., $<$) $\alpha = .05$, but not significant at (i.e., $>$) $\alpha = .01$.

(b) is FALSE: See pages 6.1-11 and 6.1-12 in the Lecture Notes (and also Appendix / A3.1).

(c) is FALSE: The probability of correctly accepting a true null hypothesis is called the confidence level of the test. The power of the test is the probability of correctly rejecting a false null hypothesis.

(d) is TRUE: If two events $A$ and $B$ are disjoint, then $A \cap B = \emptyset$, so that $P(A \cap B) = 0$. However, independent events must satisfy $P(A \cap B) = P(A)P(B)$. Thus it would have to be true that $P(A)P(B) = 0$, which is impossible if neither event $A$ nor $B$ has zero probability.

8. (a)

By definition, $r^2 = \frac{SS_{\text{Reg}}}{SS_{\text{Tot}}} = 1 - \frac{SS_{\text{Err}}}{SS_{\text{Tot}}}$, and $s_y^2 = \frac{SS_{\text{Tot}}}{n-1}$. Combining these identities yields $SS_{\text{Err}} = (1 - r^2)(n-1)s_y^2 = (1 - 0.64)(16 - 1)(5) = 27.0$. 


9. (a)

From the given information, we have “Weight” \( X \sim N(\mu, \sigma) \), with

- \( P(X < 533) = .97 \)
- \( P(X < 520) = .96 \).

However, we also know that

- \( P(Z < 1.88) = .97 \)
- \( P(Z < 1.75) = .96 \).

Hence, via \( Z = \frac{X - \mu}{\sigma} \), we have, respectively,

- \( 1.88 = \frac{533 - \mu}{\sigma} \)
- \( 1.75 = \frac{520 - \mu}{\sigma} \).

Solving these two equations simultaneously yields \( \mu = 345 \) and \( \sigma = 100 \). Therefore,

- \( P(X < 389) = P \left( Z < \frac{389 - 345}{100} \right) = P(Z < 0.44) = 0.67003 \), or 67%.

10. (b)

Water flows if either A or B is open, therefore

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.3 - (0.4)(0.3) = 0.58
\]