1. \( P(\text{Success}) = \pi = .0001, \quad P(\text{Failure}) = 1 - \pi = .9999 \)

(a) With \( n = 2000 \), the mean number of successes is \( \mu = n\pi = (2000)(.0001) = 0.2 \) pearl-bearing oysters.

(b) \( P(X = 1) = \)

\[
\text{Binomial: } \binom{2000}{1} (.0001)^1 (.9999)^{2000-1} = 0.2 (.9999)^{1999} \quad \text{or, since this is a rare event,}
\]

\[
\text{Poisson: } \frac{e^{-0.2} (0.2)^1}{1!} = e^{-0.2} (0.2)
\]

(c) Either method yields 0.1638, to four decimal places.

(d) \( P(X \geq 1) = 1 - P(X = 0) = \)

\[
\text{Binomial: } 1 - \binom{2000}{0} (.0001)^0 (.9999)^{2000-0} = 1 - (.9999)^{2000}
\]

\[
\text{Poisson: } 1 - \frac{e^{-0.2} (0.2)^0}{0!} = 1 - e^{-0.2}
\]

Either method yields 0.1813, to four decimal places.

(e) It doesn’t matter how many oysters have already been opened. Under the assumption of independence of the binary outcomes between oysters, the probability of success per individual trial remains constant at \( \pi = .0001 \). (Think of getting a run of tails on consecutive coin tosses. The next toss still has the same probability of landing on heads as the very first toss, .0001 in this case. The mistaken belief that the probability will somehow change from one independent toss to the next is known as gambler’s fallacy, and is one reason that casinos are in business, at the expense of many people who lose their shirts.)
2. (a) \( P(\text{Small}) = P(4 \leq X \leq 8) = \frac{1}{2} (8 - 4)(0.1) = 0.2 \) (area of a triangle, height = 0.1)

\( P(\text{Medium}) = P(8 \leq X \leq 12) = (12 - 8)(0.1) = 0.4 \) (area of a rectangle, height = 0.1)

\( P(\text{Large or Giant}) = P(12 \leq X \leq 20) = \frac{1}{2} (20 - 12)(0.1) = 0.4 \) (area of a triangle, height = 0.1)

The graph is clearly non-negative, and the total area = 1.

(b) \( P(\text{Giant}) = P(16 \leq X \leq 20) = \frac{1}{2} (20 - 16)(0.05) = 0.1 \) (area of a triangle, height = \( \frac{0.1}{2} = 0.05 \))

(c) \( P(\text{Large}) = P(\text{Large or Giant}) - P(\text{Giant}) = 0.4 - 0.1 = 0.3 \) (NOT a triangle!)

(d) \( P(\text{Giant} | \text{Large or Giant}) = \frac{P(\text{Giant})}{P(\text{Large or Giant})} = \frac{0.1}{0.4} = 0.25 \)

(e) We already know that \( P(4 \leq X \leq 8) = 0.2 \), from part (a). Hence, the median value must lie in the interval \( 8 \leq X \leq 12 \), and comprise below it \( 0.5 - 0.2 = 0.3 \) of the 0.4 area contained therein. Because the distribution there is uniform, it follows that the median is thus equal to \( 8 + \frac{3}{4} (12 - 8) = 8 + 3 = 11 \) inches. (See diagram above.)

(f) The expected value, i.e., the mean \( \mu \), acts as a “balance point” for the total mass. Because this distribution is skewed to the right, it follows that the mean > median, but not so far as the last interval. Therefore we might expect a randomly selected individual to be in the “Large” category.
3. \( X = \text{“Temperature (°F) at noon”} \sim N(60.0, 7.8) \)

We wish to test \( H_0: \mu = 60 \) versus the two-sided \( H_A: \mu \neq 60 \), at the \( \alpha = .05 \) significance level.

(a) In order to have 95% confidence (i.e., \( 1 - \alpha = .95 \), so that \( \alpha = .05 \), hence \( z_{.025} = 1.96 \)) to correctly retain \( H_0: \mu = 60 \) if it is true, and 99% power (i.e., \( 1 - \beta = .99 \), so that \( \beta = .01 \), hence \( z_{.01} = 2.33 \)) to correctly reject \( H_0: \mu = 60 \) (in favor of the specific alternative \( H_A: \mu = 62 \)) if it is false, we must have a minimum sample size of

\[
 n \geq \left( \frac{1.96 + 2.33}{\Delta} \right)^2, \quad \text{where} \quad \Delta = \frac{|62 - 60|}{7.8}.
\]

This computation yields \( n \geq 279.9 \), so the study requires at least 280 measurements.

(b) With \( n = 365 \) measurements, the 95% margin of error = \( (1.96)(7.8 / \sqrt{365}) = 0.8 \), so the 95% “acceptance region” for \( H_0: \mu = 60 \) is the interval (60 – 0.8, 60 + 0.8) = (59.2, 60.8)°F.

(c) The margin of error for the confidence interval is the same as for the acceptance region, i.e., 0.8. Therefore, with \( \bar{x} = 61.0°F \), the 95% confidence interval = (61 – 0.8, 61 + 0.8) = (60.2, 61.8)°F.

(d)
- Reject \( H_0 \) / Retain \( H_0 \) ?
- 95% Acceptance Region: (59.2, 60.8)°F for \( H_0: \mu = 60 \) does not contain \( \bar{x} = 61.0°F \)
- 95% Confidence Interval: (60.2, 61.8)°F does not contain the null value \( H_0: \mu = 60°F \)

(e) Therefore, this experiment suggests that there is indeed a statistically significant difference (specifically, an increase) in the current mean temperature from previous years, based on the data.

(f) \( H_0: \mu \leq 60 \) ↦ In order to show statistical significance, the hope is to reject the null hypothesis…
\( H_A: \mu > 60 \) ↦ …in favor of the alternative, so the researcher’s expectation that \( \mu > 60 \) goes here.

\[ X = \text{Temp. (°F)} \]

IF the null hypothesis is false, it would take at least 280 sample measurements to correctly reject it in favor of the particular alternative shown, 99% of the time.
4. **Given:** $X$ = “Annual number of hours worked” is normally distributed, with mean $\mu = 2080$ hours.

Sample data: $n = 9$, $\bar{x} = 2442$ hours, $s = 375$ hours

As the population standard deviation $\sigma$ is unknown, and $n < 30$, the $t$-distribution on $df = n - 1 = 8$ applies. The standard error is thus estimated by $\hat{s} = s / \sqrt{n} = 375 / \sqrt{9} = 125$ hours.

(a) 95% margin of error = $(t_{8,.025})(s / \sqrt{n}) = (2.306)(125) = 288.25$ hours

$\therefore$ 95% confidence interval for $\mu$ is $(2442 - 288.25, 2442 + 288.25) = (2153.75, 2730.25)$ hours

(b) $p$-value = $2P(T_8 \geq \frac{2442 - 2080}{125}) = 2P(T_8 \geq 2.896) = 2(0.01) = 0.02$

(c)  
- Reject $H_0$ / Retain $H_0$?
- 95% Confidence Interval: $(2153.75, 2730.25)$ for $H_0: \mu = 2080$ does not contain $\bar{x} = 2442$
- 95% $p$-value: $0.02 < 0.05 = \alpha$

(d) There is a statistically significant difference (specifically, higher) between the mean annual number of hours worked by professionals $\mu$, and the overall population mean of 2040 hours in this agency.

(e) If normality is in doubt, then a nonparametric test, such as the Wilcoxon Signed Rank test, would be appropriate.