1. (b) \( X = \# \text{ Days} \) is a discrete random variable; the average is given by the formula \( \mu = \sum x f(x) = (1)(.30) + (2)(.25) + (3)(.20) + (4)(.15) + (5)(.10) = 2.5 \text{ days}. \) Exercise: What would the variance \( \sigma^2 \) be?

2. (c)

<table>
<thead>
<tr>
<th>( X )</th>
</tr>
</thead>
<tbody>
<tr>
<td>WIN</td>
</tr>
<tr>
<td>Lose</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.70</td>
</tr>
<tr>
<td>.65</td>
</tr>
</tbody>
</table>

There are nine possible squares, each with an equally likely probability of lighting (1/9). Four are black, so the probability of winning 70 cents is 4/9. Likewise, five squares are white, so the probability of losing 65 cents is 5/9. Hence the expected value per play is given by the mean 

\[
\mu = (.70)\left(\frac{4}{9}\right) + (-.65)\left(\frac{5}{9}\right) = -.05
\]

i.e., lose 5 cents. This is essentially how casinos stay in business.

3. (b)

With a discrete random variable, \( f(x) \) represents the probability mass function (pmf), hence must satisfy two criteria: each \( f(x) \) must be between 0 and 1 – which eliminates choices (c) and (d) – and they must sum to 1 – which eliminates choice (a). Choice (b) however, satisfies both of these criteria. (The fact that the random variable \( X \) contains negative values is fine; perhaps these are temperature readings or monetary transactions, as in the previous problem, for example.)

4. (b)

We are told that this is a (continuous) density curve, hence its total area must be equal to 1. But this is a triangle, whose area is simply \( \frac{1}{2} \) (base)(height). Thus, denoting the unknown height by \( h \), we have \( \frac{1}{2} (5 - 0)(h) = 1 \), or \( h = 0.4 \). Hence the cumulative probability \( P(X < 3) \) is equal to the triangular area \( \frac{1}{2} (3 - 0)(0.4) = 0.6 \), or 60%.

5. (c)

We here have an event with probability \( \pi = 10^{-6} = 0.000001 \) – i.e., extremely rare – and a sample size of \( n = 10^6 = 1000000 \) – i.e., extremely large – which practically screams Poisson distribution, where

\[
P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}.
\]

The mean is \( \lambda = n \pi = (10^6)(10^{-6}) = 1 \) – i.e., the “expected value” is literally one win in a million – and it occurs with probability

\[
P(X = 1) = \frac{e^{-1}1!}{1!} = e^{-1} = 0.3679.
\]

6. (a)

Similarly, \( P(X = 0) = \frac{e^{-1}0!}{0!} = e^{-1} = 0.3679 \) as well! (Remember: 0! = 1 by convention)

7. (c)

Recall that if \( X \) and \( Y \) are two populations, then \( \text{Mean}(X - Y) = \text{Mean}(X) - \text{Mean}(Y) \); furthermore, if they are independent as well, then \( \text{Var}(X - Y) = \text{Var}(X) + \text{Var}(Y) \). Thus, we have \( \text{Mean}(X - Y) = 11 - 10 = 1 \text{ mm} \), and \( \text{Var}(X - Y) = 4^2 + 3^2 = 25 \text{ mm}^2 \), so that \( X - Y \sim N(1, 5) \). Therefore...

\[
P(X < Y) = P(X - Y < 0) = P\left( Z < \frac{0 - 1}{5} \right) = P(Z < -0.2) = 0.42074.
\]
8. (b) From any corner, there are 3 possible directions the spider can choose to move: along the $X$-axis (i.e., from back to front), $Y$-axis (i.e., from left to right), and $Z$-axis (i.e., straight up from floor to ceiling). In order to travel from the back floor corner to the front ceiling corner, the spider must move along each of these three axes exactly once... in any order. Hence there are “three factorial” combinations, i.e., $3! = 3 \times 2 \times 1 = 6$ possible ways. [Note: To be very explicit, the sample space consists of the six ordered triples $\{ (X, Y, Z), (X, Z, Y), (Y, X, Z), (Y, Z, X), (Z, X, Y), (Z, Y, X) \}$.]