1. (b) 

From the Central Limit Theorem, we know that if $X \sim N(\mu, \sigma)$, then $\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$.

Therefore, $\bar{X} \sim N\left(9, \frac{0.24}{\sqrt{55}}\right)$, so the $\bar{X}$-score $= 9.1$ transforms to a $Z$-score $= \frac{9.1 - 9.0}{0.24/\sqrt{55}} = 3.09$, and likewise, the symmetric $\bar{X}$-score $= 8.9$ transforms to a $Z$-score $= -3.09$. Hence, $P(8.9 \leq \bar{X} \leq 9.1) = P(-3.09 \leq Z \leq +3.09) = P(Z \leq +3.09) - P(Z \leq -3.09) = 0.99900 - 0.00100 = 0.998$, or **99.8%**.

2. (c) 

The midpoint of the 95% confidence interval $(30.2, 49.8)$ yrs is $\bar{x} = 40$ yrs, and the corresponding **95% margin of error = 9.8 yrs**, the interval’s half-width. But by definition, this is also equal to the product of the “.025 critical value” times the “standard error,” i.e., 

$(z_{0.025})(s.e.) = (1.96)(s.e.) = 9.8$, so that $s.e. = 5$ yrs. Hence the 97% margin of error $= (z_{0.015})(s.e.) = (2.17)(5)$ yrs $= 10.85$ yrs, making the 97% confidence interval $= (40 - 10.85, 40 + 10.85) = (29.15, 50.85)$ yrs.

3. (d) 

$H_0$ can be rejected at the $\alpha = .05$ level, therefore it must follow that the $p$-value $< .05$. But this gives absolutely **no information** about its relation to $\alpha = .01$.

4. (a) 

Recall that, because any $t$-distribution has heavier tails than the standard normal distribution, it follows that the $t$-score $> z$-score, for the same amount of upper tail area. Hence, when testing a null hypothesis at some fixed significance level, the $t$-test margin of error $> z$-test margin of error, i.e., the $t$-test confidence interval is **wider** than the $z$-test confidence interval. This increase in variability implies that a general null hypothesis will be rejected **less** often than not. (In other words, the null value that might be outside a $z$-test confidence interval might find itself inside a wider $t$-test confidence interval.) Hence it “requires more” of a sample result to show statistical significance with a $t$-test than with a $z$-test, i.e., it is **more** conservative.

5. (c) 

If $p = .03 < .05$, then the null hypothesis is **rejected** at the $\alpha = .05$ level, so choice (a) is eliminated. If the test is one-sided, then the left-tailed or right-tailed area (depending on whether the alternative hypothesis is “less than” or “greater than,” respectively) is equal to 0.3. Since the $p$-value of a two-sided test is the sum of both tail areas, it follows that $p = .06 > .05$, so the null hypothesis **cannot be rejected** at the $\alpha = .05$ level.
6. (d) Since a one-sided hypothesis test yields a $p$-value = .60, it follows that the standard normal distribution is divided into two areas: one of .60 (in the direction of the alternative hypothesis), and the other .40 (in the direction of the null hypothesis). Hence the $p$-value of the two-sided test (i.e., $H_0: \mu = \mu_0$ vs. $H_a: \mu \neq \mu_0$) corresponding to this alternative would be the sum of the two symmetric tail areas of .40, or .80. (Draw a picture!)

7. (e) By definition, the 95% acceptance region of a null hypothesis $H_0: \mu = \mu_0$ is the interval about the null value $\mu_0$, which is expected to contain random sample means $\bar{X}$ with 95% probability, so (b) is certainly true. Moreover, the associated margin of error is equal to the half-width of this interval (i.e., the distance from the center $\mu_0 = 45$ to either endpoint), so (c) is true as well.

8. (a) Recall that, by definition, the $p$-value is the probability that a random sample mean $\bar{X}$ is as -- or more -- extreme (i.e., as far -- or farther -- from the null value $\mu_0$) than the value of $\bar{x}$ we actually obtained... IF (i.e., given) the null hypothesis is true. Therefore if the $p$-value is small (specifically, less than the significance level $\alpha$), this indicates that there is very little agreement between the null hypothesis and our sample. Thus we have sufficient evidence to reject the null hypothesis, at level $\alpha$. With $p < .0001$, the rejection would thus be extremely strong, at any reasonable significance level $\alpha$.

9. (d)

10. (b) In the United States criminal justice system, “the defendant is presumed innocent” can be taken as a null hypothesis. If, as in this case, it is false, but is failed to be demonstrated by the evidence, then a false null hypothesis is not rejected. This is the very definition of a Type 2 error. A Type1 error is the event that a true null hypothesis is rejected, e.g., the defendant is in reality innocent, but the prosecution convinces the jury that (s)he did commit the crime. There is no such thing as Type 3 or Type 4 error. See page 1.3-1 in the Lecture Notes.

11. (f) These are all TRUE!