Discussion 3: Probability, Expected Value (or Mean) and Variance

Probability

1. (From "The Analysis of Biological Data" by Whitlock & Schulter, p. 108) Smoking and high blood pressure are risk factors for strokes and other vascular diseases. In the U.S., 17% of adults smoke and 22% have high blood pressure. Research suggests smoking and high blood pressure are independent. What is the probability a random adult has both risk factors?

2. Suppose Bureau of Labor statistics indicated that the proportions of employees in the U.S. in a particular year broke down as follows:

	biological male	biological female	total
management	0.180	0.185	0.365
non-management	0.357	0.278	0.635
total	0.537	0.463	1.000

   An employed person is chosen randomly. Given the person has a management job, what is the probability the person is a biological female? Hint: Use $P(A|B) = \dfrac{P(A \text{ and } B)}{P(B)}$.

3. (From "Statistics for the Life Sciences" by Samuels, Whittmer, and Schaffner, pp. 92-93) A medical test is used to decide whether a patient has a disease. Either the patient has the disease (*sick*) or not (*well*). Either the patient tests positive (+) or negative (−).

   The test has a 95% chance of a positive result on a sick patient (the test's *sensitivity*) and a 90% chance of a negative result on a well patient (the test's *specificity*). 8% of the population has the disease. Two mistakes can occur: a positive test on a patient who does not have the disease is called a *false positive*; and a negative test on a patient who has the disease is called a *false negative*.

   (a) Draw a probability tree, splitting first on *sick* or *well* and then on testing + or −.

   (b) Find the probability a randomly chosen person will test positive, that is $P(+)$.

   (c) Find the conditional probability a person who tests positive is sick, that is $P(\mathit{sick}|+)$.

Expected Value (or Mean) and Variance: Definitions and Properties

A canoe trip outfitter supplies rental gear.

1. When a customer rents a sleeping bag, it is randomly chosen from a pile of bags whose weights are described by this probability mass function:

sleeping bag model	weight (pounds)	probabilty
Parrot	1	0.2
Robin	3	0.4
Ptarmigan	4	0.4

   (a) Find the average weight of a rental bag.

   (b) Find the standard deviation of the weight of a bag.

2. Here are the mean and standard deviation weights of other gear rented by the outfitter:

item	μ (pounds)	σ (pounds)
sleeping pad	3	1
cooking gear	12	2
tent	8	1

   A backpack will be filled with two sleeping pads, one set of cooking gear, and one tent, each randomly and independently selected from the corresponding population of rental gear.

   (a) Which equation fits this situation better?

   - $B = 2S + C + T$, where B = backpack weight, S = sleeping pad weight, C = cooking gear weight, and T = tent weight, or
   - $B = S_1 + S_2 + C + T$, where S_1 = first sleeping pad weight and S_2 = second sleeping pad weight.

   (b) Find the average weight of a backpack.

   (c) Find the standard deviation of the weight of a backpack.