STAT 371 Exam 1

NetID (mine is “jgillett” from “jgillett@wisc.edu”): ____________

Last name: ____________________________  First name: ____________________________

Discussion (check one):

___ 341 We 9:55 in L185 Education with Chen, Crystal and Li, Jinglan
___ 342 We 1:20 in 1156 Mechanical Engineering with Chen, Crystal and Yu, Zhongjie
___ 343 We 2:25 in 357 Soils with Chen, Crystal and Yu, Zhongjie

Instructions.

1. Do not open the exam until I say “go.”

2. Put away everything except a pencil, a calculator, and your one-page (two sides) notes sheet.

3. Attempt all questions.

4. Show your work clearly. Correct answers without enough work may receive no credit.

5. Find the needed table(s) at the end of the packet. You may tear the tables sheet(s) free.

6. If a question is ambiguous, resolve it in writing. We will consider grading accordingly.

7. The exam will end when I call time. If you continue writing after I call time, you risk a penalty. (The alternative, that you get more time than your peers, is unfair.)

8. You are welcome to turn your exam in to me before I call time. However, if you are still here in the last five minutes, please remain seated until I’ve picked up all the exams.

9. Good luck!

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
<th>Your Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>15</td>
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<tr>
<td>Q2</td>
<td>10</td>
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<td>Q3</td>
<td>20</td>
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<td>Q4</td>
<td>10</td>
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<td>Q5</td>
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<td>Q6</td>
<td>10</td>
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<tr>
<td>Q7</td>
<td>20</td>
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<tr>
<td>Total</td>
<td>100</td>
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</tbody>
</table>

1
1. (15 points) Here are the numbers of blueberries on 8 randomly-chosen bushes on Flattop Mountain: 83 97 92 87 90 88 82 85. Regarding these data:

(a) find the sample mean,

**ANSWER:**
$$\bar{x} = 88 \text{ (from a calculator)} \text{ or } \bar{x} = \frac{83+97+92+87+90+88+82+85}{8} = 88.$$ 

(b) find the sample standard deviation (you do not need to show your work for this part),

**ANSWER:**
$$s \approx 4.957 \text{ (from a calculator)}$$

(c) and find the sample median.

**ANSWER:**
Here are the data after sorting: 82 83 85 87 88 90 92 97.
The sample size, \(n = 8\), is even, so the median is the average of the numbers at positions \(\frac{8}{2} = 4\) and \(4 + 1 = 5\), that is, \(M = \frac{1}{2}(87 + 88) = 87.5\).

2. (10 points) A carnival game rewards its player with one of three prizes whose values in dollars, \(x\), and associated probabilities, \(p(x)\), are given by the following probability mass function:

<table>
<thead>
<tr>
<th>(x)</th>
<th>1</th>
<th>2</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p(x))</td>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{4})</td>
<td>(\frac{1}{4})</td>
</tr>
</tbody>
</table>

(a) The mean, or expected value, of a random prize, \(X\), is \(\mu_X =

**ANSWER:**
$$\sum_x x \cdot p(x) = 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} + 8 \cdot \frac{1}{4} = 3$$

(b) The variance of a random prize, \(X\), is \(\sigma^2_X =

**ANSWER:**
$$\sum_x (x - \mu_X)^2 \cdot p(x) = (1 - 3)^2 \cdot (1/2) + (2 - 3)^2 \cdot (1/4) + (8 - 3)^2 \cdot (1/4) = 8.5$$
3. (20 points) A child’s monthly allowance in dollars, $A$, is decided in the following way. First, a fair coin is flipped. If heads comes up, the allowance is $5. If tails comes up, then a fair six-sided die is rolled, and the allowance is the number of dots facing up on the die. Make a table showing the probability mass function of $A$. (No graph is necessary.)

**ANSWER:**

The possible values of $A$ are 1, 2, 3, 4, 5, 6. Here are the associated probabilities.

\[
P(A = 1) = P(\text{coin is tails) and (die is 1})
\]
\[= P(\text{coin is tails}) \times P(\text{die is 1}) \text{ (by independence of coin and die)}
\]
\[= \frac{1}{2} \times \frac{1}{6}
\]
\[= \frac{1}{12}
\]

\[
P(A = 2) = P(A = 3) = P(A = 4) = P(A = 6) = \frac{1}{12} \text{ by similar reasoning.}
\]

\[
P(A = 5) = P(\text{(coin is heads) or ((coin is tails) and (die is 5)))}
\]
\[= P(\text{coin is tails}) + P(\text{(coin is tails) and (die is 5)})
\]
\[\text{(because } P(\text{event}) \text{ is sum of probabilities of its outcomes)}
\]
\[= \frac{1}{2} + \frac{1}{2} \times \frac{1}{6}
\]
\[= \frac{7}{12}
\]

To summarize, here is the probability mass function:

<table>
<thead>
<tr>
<th>$a$</th>
<th>$p(a)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>\frac{1}{12}</td>
</tr>
<tr>
<td>2</td>
<td>\frac{1}{12}</td>
</tr>
<tr>
<td>3</td>
<td>\frac{1}{12}</td>
</tr>
<tr>
<td>4</td>
<td>\frac{1}{12}</td>
</tr>
<tr>
<td>5</td>
<td>\frac{7}{12}</td>
</tr>
<tr>
<td>6</td>
<td>\frac{1}{12}</td>
</tr>
</tbody>
</table>
4. (10 points) One meter (m) is 39.3701 inches (in). Consider a random sample of U.S. women’s
heights with sample mean $\bar{x}_m = 1.618$ m and sample variance $s_m = 0.0025$ m$^2$.

(a) If the heights are converted to inches, what is their sample mean (in inches)?

ANSWER:

$\bar{x}_{\text{in}} = (39.3701)\bar{x}_m = (39.3701)(1.618) = 63.70082$ in

(b) If the heights are converted to inches, what is their sample variance (in inches$^2$)?

ANSWER:

$s^2_{\text{in}} = (39.3701)^2s_m^2 = (39.3701)^2(0.0025) = 3.875012$ in$^2$

5. (15 points) Suppose a baby is equally likely to be a boy or a girl, and that its sex is independent
of the sex of other children. What is the probability that a couple with four children has the
same number of boys and girls?

ANSWER:

Let $X =$ the number of girls in a randomly-chosen couple with four children. Then $X \sim \text{Bin}(4, \frac{1}{2})$ and $P($same number of boys and girls$) = P(X = 2)$ (since $X = 2$ girls implies $4 - 2 = 2$ boys) $P(X = 2) = \frac{4!}{2!(2)!}(\frac{1}{2})^2(1 - \frac{1}{2})^{4-2} = \frac{3}{8} = .375$.

(An alternate solution involves drawing a probability tree, whose levels are “first child,”
“second,” “third,” and “fourth,” to find all 16 possible outcomes of the sexes of four children:
GGGG, GGGB, GGBG, GGBB, GBGG, GBGB, GBBG, BGGG, BGGB, BGBG, BGGB, BGBB, BBGG, BBGB, BBBG, BBBB. Then count how many have two girls and two boys,
which is 6, so the probability is $\frac{6}{16} = \frac{3}{8}$.)
6. (10 points) A hospital cafeteria collected statistics on the number of calories provided by each item it offers. Here’s a fragment of those statistics:

<table>
<thead>
<tr>
<th>food</th>
<th>mean number of calories</th>
<th>standard deviation of number of calories</th>
</tr>
</thead>
<tbody>
<tr>
<td>bowl of corn flakes</td>
<td>100</td>
<td>2</td>
</tr>
<tr>
<td>container of yogurt</td>
<td>90</td>
<td>3</td>
</tr>
<tr>
<td>glass of orange juice</td>
<td>110</td>
<td>6</td>
</tr>
</tbody>
</table>

Consider patients whose breakfast consists of a bowl of corn flakes, a container of yogurt, and a glass of orange juice (randomly and independently chosen from the cafeteria’s respective supply populations).

(a) Find the mean number calories in such a breakfast.

**ANSWER:**
\[ E(\text{corn flakes + yogurt + juice}) = E(\text{corn flakes}) + E(\text{yogurt}) + E(\text{juice}) = 100 + 90 + 110 = 300. \] (The unit is calories.)

(b) Find the standard deviation of the number of calories in such a breakfast.

**ANSWER:**
First find the variance, and then take its square root to get the standard deviation.
\[ VAR(\text{corn flakes + yogurt + juice}) = VAR(\text{corn flakes}) + VAR(\text{yogurt}) + VAR(\text{juice}) = 2^2 + 3^2 + 6^2 = 49, \] so the standard deviation is \( \sqrt{49} = 7 \). (The unit is calories.)
7. (20 points) Babies born in singleton births in the U.S. have approximately normally distributed weights with mean 3.3 kg and standard deviation 0.6 kg. Regarding this population:

a. What is the probability of a baby weighing more than 5 kg?

ANSWER:
Let $X$ = weight of randomly-chosen baby. Then $X \sim N(3.3, 0.6^2)$. $P(X > 5) = P\left(\frac{X - \mu}{\sigma} > \frac{5 - 3.3}{0.6}\right) \approx P(Z > 2.83) = 1 - P(Z < 2.83) = 1 - 0.9977 = 0.0023.$

b. What is the probability of a baby weighing between 3 and 4 kg?

ANSWER:
$P(3 < X < 4) = P\left(\frac{3 - 3.3}{0.6} < \frac{X - \mu}{\sigma} < \frac{4 - 3.3}{0.6}\right) \approx P(-0.50 < Z < 1.17) = P(Z < 1.17) - P(Z < -0.50) = 0.8790 - 0.3085 = 0.5705.$

c. What is the weight of a baby in the 5th percentile? That is, find the weight such that 5% of babies weigh less.

ANSWER:
Let $x$ be the required weight. $P(X < x) = 0.05 \implies P\left(\frac{X - \mu}{\sigma} < \frac{x - 3.3}{0.6}\right) = 0.05 \implies P(Z < z) = 0.05$, where $z = \frac{x - 3.3}{0.6}$. From the table, $z \approx -1.65$ (the average of -1.64 and -1.65), so we have $-1.645 = \frac{x - 3.3}{0.6} \implies x = -1.645(0.6) + 3.3 = 2.313$. 5% of babies weigh less than 2.313 kg.

d. Suppose babies lighter than the 5th percentile require special treatment for low weight. Suppose, further, that a hospital typically has ten newborn babies per day that may be regarded as randomly and independently chosen from the population. What is the probability that two of a day’s ten babies are lighter than the 5th percentile?

ANSWER:
Let $B =$ the number of babies out of ten lighter than the 5th percentile. Then $B \sim Bin(n = 10, \pi = 0.05)$ and $P(B = 2) = \binom{10}{2} 0.05^2 (1 - 0.05)^{10-2} \approx 0.075.$