STAT 371 Exam 1 NetID (mine is "jgillett" from "jgillett@wisc.edu"): ____________

Last name: ______________________________ First name: ________________________

Discussion (check one):

Lecture 002 TuTh 9:30 in SMI 331

____ 321 Tu 11:00 in Microbial Sciences with Jia, Shengji and Cao, Luxi

____ 322 Tu 12:05 in Engineering Hall 3032 with Jia, Shengji and Cao, Luxi

____ 323 We 8:50 in Social Sciences 6102 with Teo, Benjamin and Pritchard, Nathaniel

Lecture 003 TuTh 1:00 in Soils 270

____ 331 We 7:45 in Van Hise 494 with Fogg, John and Gao, Yue

____ 332 We 12:05 in Educational Sciences 212 with Fogg, John and Gao, Yue

____ 333 Th 9:55 in Grainger 2080 with Fogg, John and Gao, Yue

Instructions:

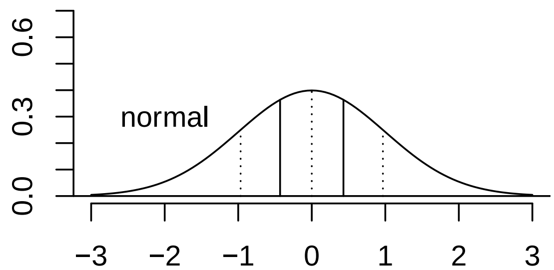
1. Do not open the exam until I say "go."
2. Put away everything except a pencil, a calculator, and your one-page (two sides) notes sheet.
3. Show your work. Correct answers without enough work may receive no credit.
4. Find the required table(s) at the end of the packet. You may tear the tables sheet(s) free.
5. If a question is ambiguous, resolve the ambiguity in writing. We will consider grading accordingly.
6. The exam ends when I call time. If you continue writing after I call time, you risk a penalty. (The alternative, that you get more time than your peers, is unfair.)
7. You are welcome to turn your exam in to me before I call time. However, if you are still here in the last five minutes, please remain seated until I've called time.

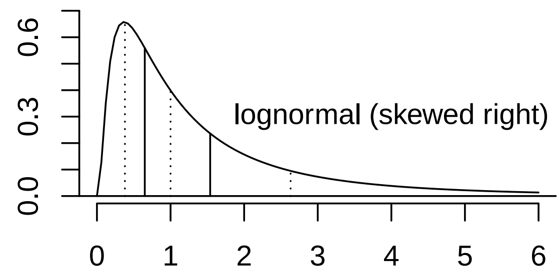
Question	Points	Deduction
Q1	10	
Q2	10	
Q3	20	
Q4	10	
Q5	10	
Q6	10	
Q7	10	
Q8	20	
Total	100	

1. Consider the random sample 5, 1, 2. We will lead you through making a normal probability plot (like the one made by `qqnorm()`) and a lognormal probability plot (unlike any you have seen) to decide from which distribution the sample seems to have come.

   (a) Sort the sample in increasing order by filling in these blanks: ______, ______, ______.
   ANSWER: 1, 2, 5 We can think of these sorted data as representatives of the left $\frac{1}{3}$, the middle $\frac{1}{3}$, and the right $\frac{1}{3}$ of the (unknown) distribution from which the sample came.

   (b) We will find corresponding representatives of $N(0, 1^2)$, drawn below on the left. We have drawn solid vertical lines that divide the distribution into three equal areas of $\frac{1}{3}$ each. We have also drawn dotted vertical lines that divide each $\frac{1}{3}$ area into two $\frac{1}{6}$ areas.





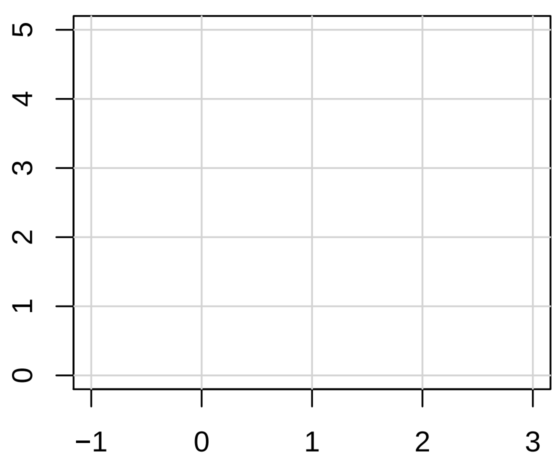
   i. The representative of the left $\frac{1}{3}$ cuts off left area $\frac{1}{6}$. Use the Z table to find it. ______
   ANSWER: -0.965

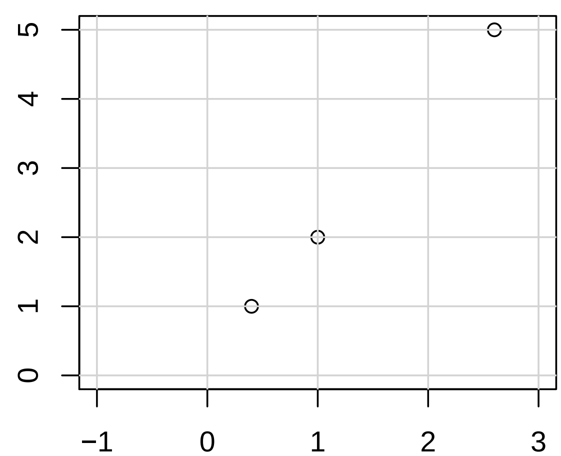
   ii. The representative of the middle $\frac{1}{3}$ cuts off left area $\frac{1}{2}$. Find its value. ______
   ANSWER: 0

   iii. The representative of the right $\frac{1}{3}$ cuts off left area $\frac{5}{6}$. Find its value. ______
   ANSWER: 0.965

   The corresponding representatives for the lognormal distribution are 0.4, 1.0, and 2.6.

   (c) Now draw the normal probability plot, below on the left, by drawing three ordered pairs of the form (x = distribution representative, y = sorted sample representative) for the left, middle, and right $\frac{1}{3}$s. We have drawn the lognormal probability plot on the right.





ANSWER: plot (-0.965, 1), (0, 2), and (0.965, 5)

(d) For which distribution do the points line up better? normal / lognormal (circle one)
ANSWER: lognormal

2. Jesse wants to invest in the stock market and considers stocks A, B, and C, each worth \$1 today. Suppose the future values of these stocks are independent.

   Let X be the value of stock A in one year. X has this probability mass function:

x	p(x)
0	0.2
1	0.4
2	0.3
5	0.1

   (a) What is the expected value of X?

   ANSWER:
   $E(X) = 0(0.2) + 1(0.4) + 2(0.3) + 5(0.1) = 1.5$

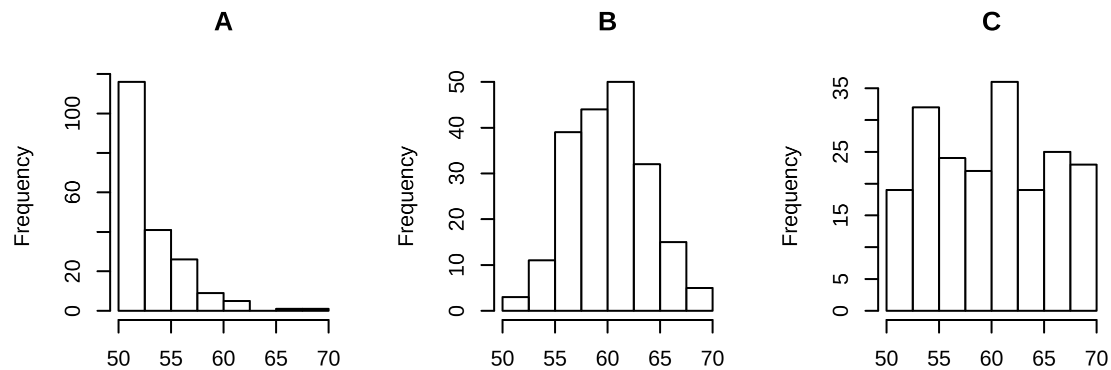
   (b) Jesse wants to quantify the risk of investing in stock A. What is the variance of X?

   ANSWER:
   $VAR(X) = (0 - 1.5)^2(0.2) + (1 - 1.5)^2(0.4) + (2 - 1.5)^2(0.3) + (5 - 1.5)^2(0.1) = 1.85$

   (c) Jesse will invest in stock A if it has high expected value and low risk. Specifically, Jesse will buy stock A if $E(X) - SD(X)$ is greater than \$1. Should he buy stock A?

   ANSWER:
   $E(X) - SD(X) = 1.5 - \sqrt{1.85} = 1.5 - 1.36 = 0.14 < 1$, so no, he should not buy it.

3. The histograms **A**, **B**, and **C** represent samples from three different populations. Hint: I solved parts (a), (b), and (c) without doing any computation–I just looked at the shapes of the histograms.



(a) In which of the three histograms is the mean clearly greater than the median?

ANSWER:
A. The median is the point with half the sample on each side (around 52?). The mean is the balance point. This histogram would not balance on its median, with half the weight on each side, because the stretched out right half would have more leverage than the compact left half, so the mean is right of the median. (The other two histograms are nearly symmetric, so each mean should be near the corresponding median.)

(b) Which of **B** and **C** has the higher standard deviation?

ANSWER:
C. The two means are about the same (62?), but **C** has many more values far from the mean (near 50 and 70) than **B**.

(c) Order **A**, **B**, and **C** by the third quartile, Q_3, from smallest to largest.

ANSWER:
A, **B**, **C**. To estimate Q_3 for each histogram, first divide each histogram into equal-area right and left halves, marking the division as the median. Then divide the right half again (into quarter), marking the division as Q_3.

(d) Consider sample **B**. Suppose that its sample mean is $\bar{b} = 60$ on a sample of size $n = 200$. Suppose that one more observation (the 201st) is taken with a value of 65. Calculate the new sample mean.

ANSWER:
60.025: The original sum is $s = \sum b_i = n\left(\frac{1}{n}\sum b_i\right) = n\bar{b} = 200(60) = 12000$. The new sum is $12000 + 65 = 12065$, so the new mean is $\frac{1}{201}(12065) \approx 60.025$.

4. An elevator can safely hold up to 36 people weighing a total of less than 7200 pounds. Suppose the weights of adult U.S. men have mean 175 and standard deviation 50 pounds. Find the probability a random sample of 36 adult U.S. men can safely use the elevator.

ANSWER:

Let $X_1, \ldots, X_{36}$ be the weights of the men in the sample. $n = 36$ is large (our rule of thumb requires $n > 30$), so the CLT says $\bar{X} \sim N(\mu, \sigma^2/n)(\approx) = N(175, 50^2/36)$.

$$\begin{aligned} P(\text{total weight of 36 men } < 7200) &= P\left(\sum X_i < 7200\right) \\ &= P\left(\frac{1}{n}\sum X_i < \frac{7200}{36}\right) \\ &= P(\bar{X} < 200) \\ &= P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < \frac{200 - 175}{50/\sqrt{36}}\right) \\ &= P(Z < 3) \\ &= .9987 \end{aligned}$$

5. In a children's carnival game, the child rolls a fair six-sided die. If a 5 or 6 shows, the child wins. Otherwise the child loses.

Seven children play the game. What is the probability exactly 2 of them win?

ANSWER:

Let $X =$ #winning children from among the 7 players $\sim \text{Bin}(7, \frac{2}{6} = \frac{1}{3})$. $P(X = 2) = \frac{7!}{2!(7-2)!}(\frac{1}{3})^2(1 - \frac{1}{3})^{7-2} \approx 0.307$

6. To ensure efficient usage of a web server, a system administrator wants to estimate the population mean number of users per minute over all daytime minutes of the week. She knows from earlier research that the population standard deviation is steady around 9. She records the numbers of users at 6 randomly selected daytimes through the week, with these results: 78, 62, 80, 57, 71, 72.

   She makes a normal QQ plot. It looks straight enough (in the context of QQ plots of random normal samples of size 6) that she is williing to assume a normal population.

   (a) Construct a 97% confidence interval for the population mean of number of users per minute. Express it as "center ± error margin." (Hint: 97% is not the usual 95%.)

   ANSWER:
   $n = 6$, $\bar{x} = 70$, and $\sigma = 9s$. The confidence level is $(100\%)(1 - \alpha) = 97\%$, so $1 - \alpha = .97 \implies \alpha = .03$ and we need and $z_{.03/2} = z_{.015} = 2.17$. The interval is $\bar{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 70 \pm 2.17\frac{9}{\sqrt{6}} \approx 70 \pm 7.97 = (62.03, 77.97)$.

   (b) What sample size is required to get an error margin of 2 for a 97% interval?

   ANSWER:
   To find the (approximate) n required for a given error margin m, set m to the error margin of the Z interval formula: $m = z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \implies n = \left(z_{\alpha/2}\frac{\sigma}{m}\right)^2 \approx \left(2.17 \times \frac{9}{2}\right)^2 = 95.36$, which we round up to $n = 96$.

7. Sammy, Matt, and Ale are working on a group project. Sammy completes group work 99% of the time, Matt completes it 95% of the time, and Ale completes it 75% of the time. Their completions are independent. What is the chance the project will not be completed?

   ANSWER:

$$
\begin{aligned}
P(\text{project not completed}) &= 1 - P(\text{project completed}) \\
&= 1 - P(\text{Sammy completes and Matt completes and Ale completes}) \\
&= 1 - P(\text{Sammy completes}) \times P(\text{Matt completes}) \times P(\text{Ale completes}) \\
&\quad (\text{ by independence}) \\
&= 1 - .99 \times .95 \times .75 \\
&\approx 0.295
\end{aligned}
$$

8. The weights (in pounds) of pumpkins in a patch are distributed as $N(15, 2^2)$.

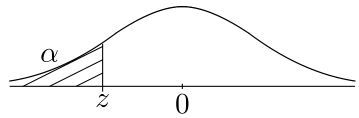
(a) What proportion of pumkins weighs more than 15 pounds?

ANSWER:
Let W be the weight of a randomly chosen pumpkin. Then $W \sim N(15, 2^2)$.
Since $N(15, 2^2)$ is symmetric about the mean 15, $P(W > 15) = \frac{1}{2}$. (Or $P(W > 15) = P\left(\frac{W-\mu}{\sigma} > \frac{15-15}{2}\right) = P(Z > 0) = P(Z < 0) = .5$.)

(b) Find the weight such that 98.5% of all pumpkins in the patch are lighter.

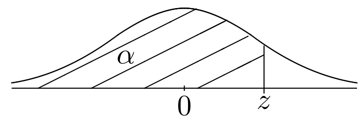
ANSWER:
Let w be the weight such that 98.5% weigh less. Then $P(W < w) = .985 \implies P((Z = \frac{W-\mu}{\sigma}) < \frac{w-15}{2}) = .985$. From the table, $z = 2.17$ cuts off left area .9850, so $\frac{w-15}{2} = 2.17 \implies w = 2.17(2) + 15 = 19.34$.
98.5% of pumkins weigh less than 19.34 pounds.

For $z = a.bc$, look in row $a.b$ and column $.0c$ to find $P(Z < z)$. e.g.
For $z = 1.42$, look in row 1.4 and column .02 to find $P(Z < 1.42) = .9222$ (on next page).



Cumulative $N(0, 1)$ Distribution, $z \leq 0$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641



Cumulative $N(0,1)$ Distribution, $z \geq 0$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999