STAT 371 Exam 1 NetID (mine is "jgillett" from "jgillett@wisc.edu"): ____________

Last name: ______________________________ First name: ________________________

Discussion (check one):

____ 321 Tu 12:05-12:55 Engineering Hall 2239 Fan, Ning and Yu, Zhongjie

____ 322 Tu 4:35-5:25 Microbial Sciences 1420 Hu, Bowen and Lin, Yunong

____ 323 We 8:50-9:50 Engineering Hall 3032 Fan, Ning and Lin, Yunong

Instructions.

1. Do not open the exam until I say "go."
2. Put away everything except a pencil, a calculator, and your one-page (two sides) notes sheet.
3. Attempt all questions.
4. Show your work clearly. Correct answers without enough work may receive no credit.
5. Find the needed table(s) at the end of the packet. You may tear the tables sheet(s) free.
6. If a question is ambiguous, resolve it in writing. We will consider grading accordingly.
7. The exam will end when I call time. If you continue writing after I call time, you risk a penalty. (The alternative, that you get more time than your peers, is unfair.)
8. You are welcome to turn your exam in to me before I call time. However, if you are still here in the last five minutes, please remain seated until I've picked up all the exams.
9. Good luck!

Question	Points	Your Score
Q1	20	
Q2	10	
Q3	20	
Q4	10	
Q5	20	
Q6	20	
Total	100	

1. The numbers of leaves in a sample of 7 maple seedlings chosen from a nursery were counted, with these results: 3, 5, 6, 6, 10, 11, 15.

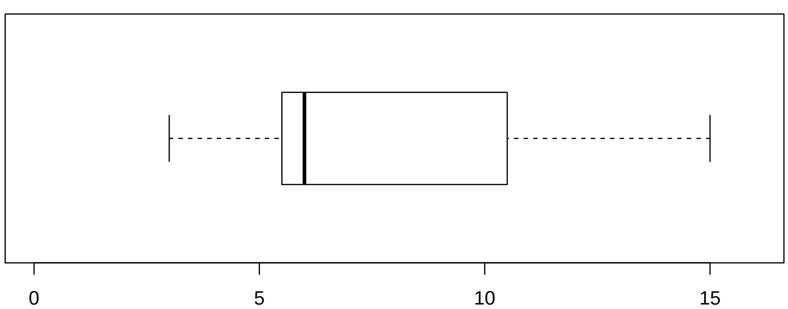
   (a) Find the sample mean number of leaves.

   ANSWER:
   $\frac{1}{7}(3 + 5 + 6 + 6 + 10 + 11 + 15) = 8$

   (b) Find the sample median number of leaves.

   ANSWER:
   $n = 7$ is odd, so the median is the central element of the sorted sample at position $\frac{7+1}{2} = 4$. That is, the median is 6.

   (c) A boxplot of these data is below. Draw a line segment on the boxplot whose length is the interquartile range of the numbers of leaves.



   ANSWER:
   The box extends from Q_1 to Q_3, so the segment should have the same length as the box.

2. Suppose canteloupe diameters are normally distributed with a mean of 6 inches and a standard deviation of 1 inch.

   (a) Find the probability a randomly-chosen canteloupe has diameter less than 6.2 inches.

   ANSWER:
   Let W = weight of a randomly chosen canteloupe. $P(W < 6.2) = P\left(\frac{W-\mu}{\sigma} < \frac{6.2-6}{1}\right) = P(Z < 0.2) = 0.5793.$

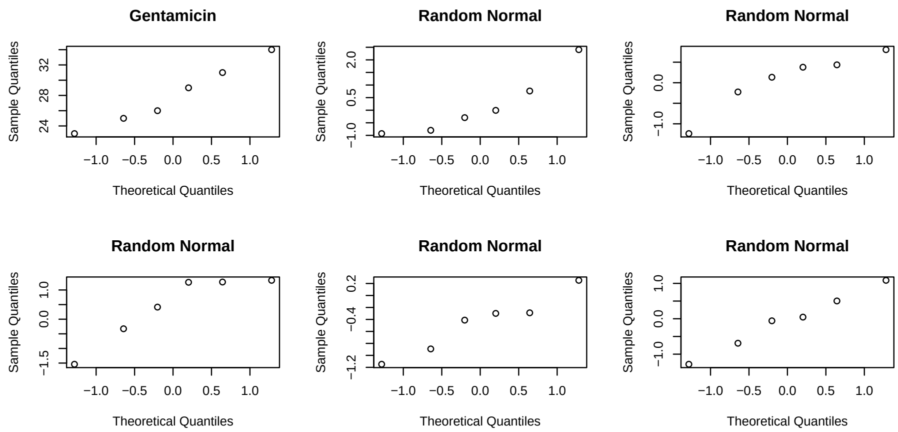
   (b) Find the probability that 100 randomly-chosen canteloupes have a sample mean diameter less than 6.2 inches.

   ANSWER:
   Let $W_1, \ldots, W_n$ be the diameters of the $n = 100$ canteloupes in the sample, and let $\bar{W} = \frac{1}{n}\sum_{i=1}^{n} W_i$ be their sample mean diameter. $n = 100$ is large enough for the CLT to apply (our rule of thumb requires $n > 30$), so $\bar{W} \sim N\left(6, \frac{\sigma^2}{n} = \left(\frac{1}{\sqrt{n}}\right)^2\right)$ $(\approx)$. Then

$$\begin{aligned} P(\bar{W} < 6.2) &= P\left(\frac{\bar{W}-\mu}{\sigma/\sqrt{n}} < \frac{6.2-6}{1/\sqrt{100}}\right) \\ &= P(Z < 2.00) \\ &= 0.9772 \end{aligned}$$

3. To test the antibiotic Gentamicin on adult Suffolk sheep on a large farm, a simple random sample of six healthy adults were injected with Gentamicin at a dosage of 10 mg/kg body weight. Their blood serum concentration (μg/ml) of Gentamicin 1.5 hours after injection were as follows: 29, 26, 34, 31, 23, 25. For these data, the sample mean is 28.0. Experience with similar experiments suggest that the population standard deviation is close to 4.6.

Here is a QQ plot of the Gentamicin data (top left), along with 5 QQ plots of random samples of the same size from a normal population for reference:



(a) Is it plausible that the population of concentrations is normally distributed?
ANSWER:
Yes, as the departure from linear in the Gentamicin plot is no more than that of several of the 6 plots of random samples from a normal distrubtion.

(b) Find a 99.8% confidence interval for the unknown population mean concentration.
ANSWER:
For 99.8% confidence we have $1 - \alpha = .998 \implies \alpha = .002$ and we need $z_{\alpha/2} = z_{.002/2} = z_{.001} = 3.10$. Our interval is $\bar{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 28.0 \pm 3.1\frac{4.6}{\sqrt{6}} = 28.0 \pm 5.8$.

(c) Mark those statements about the interval which are true. (Suppose you did the interval calculation correctly.)

_____ Lowering the confidence level from 99.8% to 95% would lower the interval width.
_____ The probability our calculated interval contains the true mean is 0.998.
_____ Increasing the sample size from 6 to 10 would increase the confidence level.
_____ If we repeated the experiment and interval calculation with a new sample of six sheep each time, about 99.8% of our intervals would contain the true mean.

ANSWER:

X Lowering the confidence level from 99.8% to 95% would lower the interval width.
_____ The probability our calculated interval contains the true mean is 0.998.
_____ Increasing the sample size from 6 to 10 would increase the confidence level.
X If we repeated the experiment and interval calculation with a new sample of six sheep each time, about 99.8% of our intervals would contain the true mean.

4. A community organization holds a raffle for \$10000 motorcycle by selling 1000 tickets for \$25 each and then randomly identifying one as the winner. You buy one ticket.

   (a) What is the expected value of your profit from the raffle?

   ANSWER:
   Your two outcomes are win and lose, with probabilities $\frac{1}{1000}$ and $\frac{999}{1000}$, respectively. If you win, your profit is $10000 - 25 = 9975$. If you lose your profit is $0 - 25 = -25$. The expected value of your profit is therefore $\frac{1}{1000}(9975) + \frac{999}{1000}(-25) = -15$.

   (b) What is the expected value of the community organization's profit from the raffle?

   ANSWER:
   The community organization's profit is \$25(1000) - \$10000 = \$15000.

5. A factory assembles each of the kick scooters it makes from a frame, a handlebar, and two wheels, all chosen randomly and independently from piles of these respective parts. (Hint: Two different wheels are chosen, not two copies of one wheel.) The parts piles have these weight properties:

part	mean weight (kg)	standard deviation of weight (kg)
frame	3	0.1
handlebar	1	0.1
wheel	0.5	0.2

(a) Find the mean weight of a scooter.

ANSWER:
Use the properties of expected value, $E()$, to say

$$\begin{aligned} E(\text{(weight of) scooter}) &= E(\text{frame} + \text{handlebar} + \text{front wheel} + \text{back wheel}) \\ &= E(\text{frame}) + E(\text{handlebar}) + E(\text{front wheel}) + E(\text{back wheel}) \\ &= 3 + 1 + 0.5 + 0.5 = 5. \text{ (The unit is kg.)} \end{aligned}$$

(b) Find the standard deviation of the weight of a scooter.

ANSWER:
First find the variance, and then take its square root to get the standard deviation.
Use the properties of variance, $VAR()$, to say

$$\begin{aligned} VAR(\text{scooter weight}) &= VAR(\text{frame} + \text{handlebar} + \text{front wheel} + \text{back wheel}) \\ &\quad \text{(since the four choices are independent)} \\ &= VAR(\text{frame}) + VAR(\text{handlebar}) + VAR(\text{front wheel}) + VAR(\text{back wheel}) \\ &= 0.1^2 + 0.1^2 + 0.2^2 + 0.2^2 = 0.1 \end{aligned}$$

The standard deviation is then $\sqrt{0.10} \approx 0.316$. (The unit is kg.)
Note: Since two different wheels are chosen, not two copies of one wheel, it is incorrect to write $VAR(\text{scooter weight}) = VAR(\text{frame} + \text{handlebar} + 2(\text{wheel})) = \ldots$.

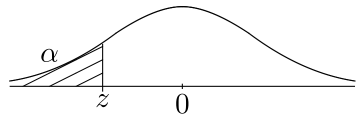
6. A new variety of turf grass has been developed for use on golf coursess, with the goal of obtaining a germination rate of 85%. To evaluate the grass, 20 seeds are planted in a greenhouse so that each seed will be exposed to identical conditions. Whether each seed germinates is independent of what happens to the other seeds. If the 85% germination rate is correct, what is the probability that 19 or more of the 20 seeds will germinate?

ANSWER:

Let X = number of 20 seeds that germinate. Then $X \sim \text{Bin}(n = 20, \pi = 0.85)$.

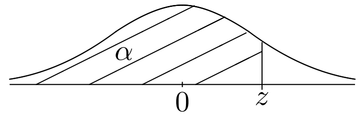
$$\begin{aligned}
P(\text{19 or more germinate}) &= P(X \geq 19) \\
&= P(X = 19) + P(X = 20) \\
&= \binom{20}{19}(0.85)^{19}(1 - 0.85)^{20-19} + \binom{20}{20}(0.85)^{20}(1 - 0.85)^{20-20} \\
&\approx 0.1368 + 0.0388 \\
&= 0.1756
\end{aligned}$$

For $z = a.bc$, look in row $a.b$ and column $.0c$ to find $P(Z < z)$. e.g.
For $z = 1.42$, look in row 1.4 and column .02 to find $P(Z < 1.42) = .9222$ (on next page).



Cumulative $N(0,1)$ Distribution, $z \le 0$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641



Cumulative $N(0,1)$ Distribution, $z \geq 0$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999