STAT 371 Exam 1 NetID (mine is "jgillett" from "jgillett@wisc.edu"): ____________

Last name: ______________________________ First name: ________________________

Discussion (check one):

____ 311 TuTh 10:20-11:10 Mechanical Engineering 1152 Brown, Jared and Huang, Kunling

____ 312 TuTh 10:20-11:10 Russel Labs 104 Hu, Bowen and Cao, Wenzhi

____ 321 TuTh 1:10-2:00 Van Hise 387 Xu, Yuqing and Huang, Kunling

Instructions.

1. Do not open the exam until I say "go."
2. Put away everything except a pencil, a calculator, and your one-page (two sides) notes sheet.
3. Attempt all questions.
4. Show your work clearly. Correct answers without enough work may receive no credit.
5. Find the needed table(s) at the end of the packet. You may tear the tables sheet(s) free.
6. If a question is ambiguous, resolve it in writing. We will consider grading accordingly.
7. The exam will end when I call time. If you continue writing after I call time, you risk a penalty. (The alternative, that you get more time than your peers, is unfair.)
8. You are welcome to turn your exam in to me before I call time. However, if you are still here in the last five minutes, please remain seated until I've picked up all the exams.
9. Good luck!

Question	Points	Your Score
Q1	20	
Q2	10	
Q3	20	
Q4	10	
Q5	20	
Q6	20	
Total	100	

1. (20 points) Suppose NBA player weights (in pounds) are $N(221, 15^2)$.

   (a) Find the weight such that 20% of players weigh less than that weight.

   ANSWER:
   Let W = weight of a randomly chosen player and x = the desired 20th percentile weight. $P(W < x) = .2 \implies P(Z < \frac{x-221}{15}) = .2 \implies \frac{x-221}{15} = -.845 \implies x = 208.325$

   (b) A random group of 5 NBA players (still with weights from $N(221, 15^2)$) cross a playground bridge together, even though its breaking strength is only 1000 pounds. What is the probability that it breaks?

   ANSWER:
   Let S = sum of their weights $= 5\bar{W}$(since $\bar{W} = \frac{1}{5}\sum_{i=1}^{5} W_i$). Since each $W_i \sim N(221, 15^2)$, $\bar{W} \sim N(221, \frac{15^2}{5})$ and $S = 5\bar{W} \sim N(5 \cdot 221, 5^2\frac{15^2}{5}) = N(1105, 33.54^2)$. Then $P(S > 1000) = P(Z > \frac{1000-1105}{33.54}) = P(Z > -3.13) = 1 - P(Z < -3.13) = 1 - .001 = .999 \approx 1.00$

2. (10 points) Here are several questions about summary statistics.

(a) Consider these summary statistics:

- IQR = interquartile range
- M = sample median
- Q_1 = first quartile
- S = sample standard deviation
- $\bar{X}$ = sample mean

Which of them is least affected by an outlier? (Circle one.)

i. IQR, M, and Q_1
ii. IQR and S
iii. M and $\bar{X}$
iv. S and $\bar{X}$
v. None of the above

ANSWER: i

(b) R was used to get summary statistics on data on the average commute time (in minutes) for each of 51 states (or, rather, 50 states and the District of Columbia):

```
commute = c(15.2, 15.4, 16.5, 16.9, 17.5, 17.5, 18.1, 18.9, 19.1, 19.4, 19.5, 19.7,
  19.9, 20.3, 20.4, 21, 21.2, 21.6, 21.7, 21.8, 21.8, 22.1, 22.1, 22.5, 22.6,
  22.7, 22.7, 22.9, 23, 23.2, 23.3, 23.3, 23.4, 23.4, 23.6, 23.7, 23.8, 24.5,
  24.6, 24.7, 24.8, 24.8, 25.8, 26, 26.1, 26.5, 27, 28.4, 28.5, 30.2, 30.4)
> summary(commute)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  15.20   20.10   22.70   22.43   24.55   30.40
>
```

What is the interquartile range (IQR) of the commute times? (Hint: You do not need to use the raw data.)

ANSWER:
IQR = $Q_3 - Q_1$ = 24.55 - 20.10 = 4.45

(c) Find the third quartile of this set of spruce log lengths (in feet):
8.7 9.2 8.7 8.0 8.5 10.1 7.5 7.8 8.8 8.0

ANSWER:
Sort: 7.5 7.8 8.0 8.0 8.5 8.7 8.7 8.8 9.2 10.1. Q_3 is the median of the second half of the (sorted) data, that is, the median of 8.7 8.7 8.8 9.2 10.1. This half has size 5, so Q_3, its median, is at position $\frac{5+1}{2} = 3$. That is, $Q_3 = 8.8$.

3. (20 points) In a sample of 100 shipping boxes, the sample mean compressive strength was 6000 N. The population standard deviation was known, from prior experience, to be 200 N.

   (a) Find a 95% confidence interval for the mean compressive strength, expressed as "center $\pm$ error margin".

   ANSWER: $6000 \pm 1.96 * 200/\sqrt{100} = 6000 \pm 39.2$

   (b) Approximately how many boxes must be sampled so that a 95% confidence interval will specify the mean to within ± 25 N?

   ANSWER: $n = (1.96 * 200/25)^2 = 245.86 \implies$ use $n = 246$

   (c) Nobel economist Daniel McFaddon worked on the Council of Economic Advisors for President Lyndon Johnson. When McFadden offered a range of forecasts for economic growth, the President replied, "Ranges are for cattle; give me one number." What confidence level is associated with a confidence interval whose error margin is 0?

   ANSWER: margin $= z_{\alpha/2}\frac{s}{\sqrt{n}} = 0 \implies z_{\alpha/2} = 0 \implies \alpha/2 = .5$ => confidence level $= 1 - \alpha = 0$

4. (10 points) Suppose each ticket in a lottery has a $\frac{1}{8}$ chance of being a winner. What is the probability of having exactly 4 winners in a randomly selected group of 10 tickets?

ANSWER:

Let X = number of winners in the 10 tickets. Then $X \sim \text{Bin}(n = 10, \pi = \frac{1}{8})$. $P(X = 4) = \binom{10}{4}(1/8)^4(1-1/8)^{10-4} = .0230$

5. (20 points) A class of students took a quiz whose score distribution is in the table below.

score	1	2	3	4
proportion of class with score	0	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{2}$

(a) What is the mean score?

ANSWER:

$\sum_x x \cdot p(x) = 1 \cdot 0 + 2 \cdot \frac{1}{8} + 3 \cdot \frac{3}{8} + 4 \cdot \frac{1}{2} = 3.375$

(b) What is the variance of the scores?

ANSWER:

$\sum_x (x - \mu_X)^2 \cdot p(x) = (1 - 3.375)^2 * (0) + (2 - 3.375)^2 * (1/8) + (3 - 3.375)^2 * (3/8) + (4 - 3.375)^2 * (1/2) = 0.484375$

6. (20 points) A grandmother will put a pile of money into two uncertain investments:

- a stock whose return has a mean of 6% and a standard deviation of 1%
- a bond whose return has a mean of 4% and a standard deviation of 0.5%

Suppose she puts half her money in the stock and half in the bond. Let R_s = the return on the stock and R_b = the return on the bond (each as a percentage). Then R = her return $= \frac{1}{2}R_s + \frac{1}{2}R_b = \frac{1}{2}(R_s + R_b)$.

(a) What is the expected value of her return?

ANSWER:
$\mu_R = \mu_{\frac{1}{2}}(R_s + R_b) = \frac{1}{2}(\mu_{R_s} + \mu_{R_b}) = \frac{1}{2}(6 + 4) = 5.$

(b) What is the standard deviation of her return?

ANSWER:
$\sigma_R^2 = \sigma^2_{\frac{1}{2}(R_s+R_b)} = \left(\frac{1}{2}\right)^2 (\sigma^2_{R_s} + \sigma^2_{R_b}) = \frac{1}{4}(1^2 + .5^2) = \frac{5}{16} = 0.3125.$
$\implies \sigma_R = \sqrt{0.3125} \approx .5590.$