STAT 371 Exam 1 NetID (mine is “jgillett” from “jgillett@wisc.edu”): ____________

Last name: ______________________________ First name: ________________________

Discussion (check one):

____ 311 Hao Chen in Sterling 1333

____ 312 Kunling Huang in Ingraham 224

Instructions:

1. Do not open the exam until I say “go.”
2. Put away everything except a pencil, a calculator, and your one-page (two sides) notes sheet.
3. Show your work. Correct answers without enough work may receive no credit.
4. Find the required table(s) at the end of the packet. You may tear the tables sheet(s) free.
5. If a question is ambiguous, resolve the ambiguity in writing. We will consider grading accordingly.
6. The exam ends when I call time. If you continue writing after I call time, you risk a penalty. (The alternative, that you get more time than your peers, is unfair.)
7. You are welcome to turn your exam in to me before I call time. However, if you are still here in the last five minutes, please remain seated until I’ve called time.

Question	Points	Deduction
Q1	10	
Q2	10	
Q3	20	
Q4	20	
Q5	20	
Q6	20	
Total	100	

1. The numbers of leaves in a sample of 7 maple seedlings chosen from a nursery were counted, with these results:

   3, 5, 6, 6, 10, 11, 15

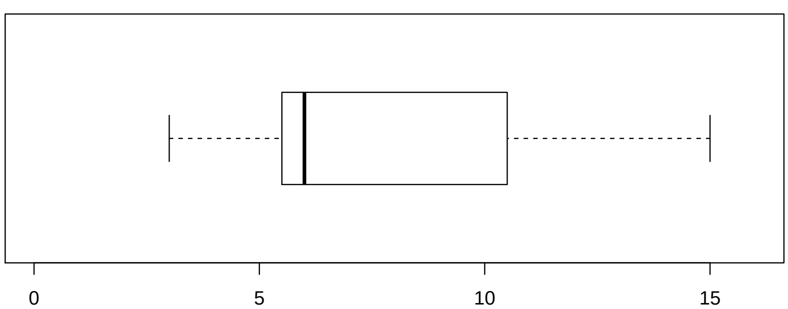
   (a) Find the sample mean number of leaves.

   ANSWER:
   $\frac{1}{7}(3 + 5 + 6 + 6 + 10 + 11 + 15) = 8$

   (b) Find the sample median number of leaves.

   ANSWER:
   $n = 7$ is odd, so the median is the central element of the sorted sample at position $\frac{7+1}{2} = 4$, that is, the median is 6.

   (c) A boxplot of these data is below. Draw a line segment on the boxplot whose length is the interquartile range of the numbers of leaves.



   ANSWER:
   The box extends from Q_1 to Q_3, so the segment should have the same length as the box.

2. A simple random sample of 6 Babcock Hall ice cream cones served in a day had weights 4.2, 4.1, 4.0, 4.2, 3.7, and 4.3 ounces. Suppose that the day's cone weights are normally distributed and experience indicates that the population standard deviation holds steady at about 0.2.

   (a) Find a 97% confidence interval for the population average weight of the day's cones. Express it in the format "interval center ± error margin."

   ANSWER:
   $n = 6$, $\bar{x} = 4.083$, and $\sigma = 0.2$. The confidence level is $(100\%)(1 - \alpha) = 97\%$, so $1 - \alpha = .97 \implies \alpha = .03$ and we need and $z_{.03/2} = z_{.015} = 2.170$. The interval is $4.083 \pm 2.170 \frac{0.2}{\sqrt{6}} = 4.083 \pm .177 = (3.906, 4.260)$ ounces.

   (b) Referring to the previous problem on ice cream cones, what does your interval mean in the context of the problem?

   ANSWER:
   If I were to draw many random samples of size 6 from the same population and calculate an interval by the same method from each sample, about 97% of those intervals would contain the population mean.
   OR
   I have 97% confidence that my interval covers the population mean weight of cones.

   (c) What (approximate) sample size is required to get an error margin of 0.10 for a 97% interval?

   ANSWER:
   To find the (approximate) n required for a given error margin m, set m to the error margin of the Z interval formula: $m = z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \implies n = \left(z_{\alpha/2} \frac{\sigma}{m}\right)^2 \approx \left(2.170 \times \frac{0.2}{.1}\right)^2 = 18.84$, which we round up to $n = 19$.

3. Of 100 kids who visited a pool concession stand, 40 bought popsicles, 30 bought gum, and 20 bought both. (So $40 - 20 = 20$ bought popsicles only and $30 - 20 = 10$ bought gum only.) If one of these kids chosen at random bought a popsicle, find the probability that child also bought gum.

ANSWER:

Let E be the event that one of these kids bought gum and F be the event that one of them bought a popsicle.

$P(E|F) = \frac{P(E \text{ and } F)}{P(F)} = \frac{20/100}{40/100} = 1/2.$

4. A fair coin is flipped, with heads counting as 1 and tails counting as 0, and a fair six-sided die is rolled. Let the random variable X be the sum of the coin flip and the die roll. (For example, if the coin is heads and the die is 3, then $x = 1 + 3 = 4$, and if the coin is tails and the die is 3, then $x = 0 + 3 = 3$.) Hint: Drawing a tree can help with articulating the possible outcomes.

   (a) Find the probability mass function of X by specifiying the values x that X can take and the probability $p(x) = P(X = x)$ of each of those values.

x							
$p(x)$							

ANSWER:

x	1	2	3	4	5	6	7
$p(x)$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{12}$

   (b) Find the expected value (or mean) of X.

ANSWER:

$\mathrm{E}(X) = 1 * (1/12) + 2 * (1/6) + 3 * (1/6) + 4 * (1/6) + 5 * (1/6) + 6 * (1/6) + 7 * (1/12) = 4$

5. A botanist measures a plant's weight on two scales. The first reports a random weight F that has mean 2 and standard deviation 1 (ounces). The second reports a random weight S that has mean 2 and standard deviation 2 (ounces) and is independent of the first measurement.

   (a) The botanist estimates the plant's weight W as $W = \frac{1}{3}(2F + S)$.

      i. Find the expected value of W.

      ANSWER:
      Use the properties of expected value, $E()$, to say $E(W) = E\left(\frac{1}{3}(2F + S)\right) = \frac{1}{3}(E(2F + S) = \frac{1}{3}\left(E(2F) + E(S)\right) = \frac{1}{3}(2E(F) + E(S)) = \frac{1}{3}(2 \cdot 2 + 2) = 2$.

      ii. Find the standard deviation of W.

      ANSWER:
      First find the variance, and then take its square root to get the standard deviation.
      $VAR(W) = VAR(\frac{1}{3}(2F + S) = \left(\frac{1}{3}\right)^2 VAR(2F + S) = \frac{1}{9}\left(VAR(2F) + VAR(S)\right) = \frac{1}{9}\left(2^2 VAR(F) + VAR(S)\right) = \frac{1}{9}(4 \cdot 1^2 + 2^2) = \frac{8}{9}$.
      $SD(W) = \sqrt{VAR(W)} = \sqrt{\frac{8}{9}} \approx .943$.

   (Note that another way to estimate the weight is to just average the two weights as $A = \frac{1}{2}(F + S)$. This method gives mean 2 and standard deviation 1.118.)

   (b) Suppose the second measurement S is normally distributed. Find the probability this second measurement is more than 4.

   ANSWER:
   $S \sim N(2, 2^2)$, so $P(S > 4) = P\left(\frac{S-\mu}{\sigma} > \frac{4-2}{2}\right) = P(Z > 1.00) = P(Z < -1.00) = .1587$.

6. Each customer at a food cart chooses a sandwich randomly by turning a spinner that has probability 1/7 of indicating a Philly cheesesteak.

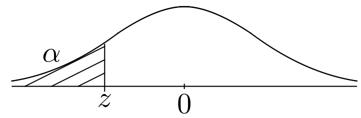
   (a) What is the probility of the next 4 customers will buy a total of 2 cheesesteaks?

   ANSWER:
   Let $X =$ #cheesesteaks bought by the next 4 customers $\sim$ Bin$(4, \frac{1}{7})$. $P(X = 2) = \frac{4!}{2!(4-2)!}(\frac{1}{7})^2(1 - \frac{1}{7})^{4-2} \approx .090$

   (b) What is the expected number of cheesesteaks bought by the next 4 customers?

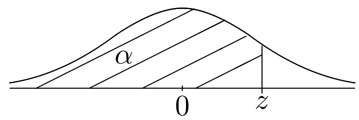
   ANSWER:
   $E(X) = n\pi = 4 \times \frac{1}{7} = \frac{4}{7} \approx .571$

For $z = a.bc$, look in row $a.b$ and column $.0c$ to find $P(Z < z)$. e.g.
For $z = 1.42$, look in row 1.4 and column .02 to find $P(Z < 1.42) = .9222$ (on next page).



Cumulative $N(0,1)$ Distribution, $z \leq 0$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641



Cumulative $N(0,1)$ Distribution, $z \geq 0$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	$\boxed{.9222}$	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999