STAT 371 Exam 1   NetID (mine is "jgillett" from "jgillett@wisc.edu"): _____

Last name: _____   First name: _____

Discussion (check one):

Lecture 002 TuTh 1:00-2:15 Soils 270

_____ 321 Tu 3:30-4:20 Grainger 1185 with Trane, Ralph and Park, Chan

_____ 322 Tu 4:35-5:25 Social Sciences 5231 with Trane, Ralph and Park, Chan

_____ 323 We 9:55-10:45 Sterling 1313 with Trane, Ralph and Liu, Hongzhi

Lecture 003 TuTh 2:30-3:45 Van Vleck B130

_____ 331 Tu 4:35-5:25 Psychology 121 with White, David and Pritchard, Nathaniel

_____ 332 We 9:55-10:45 Psychology 103 with White, David and Pritchard, Nathaniel

_____ 333 We 11:00-11:50 Ingraham 22 with White, David and Pritchard, Nathaniel

Instructions:

1. Do not open the exam until I say "go."

2. Put away everything except a pencil, a calculator, and your one-page (two sides) notes sheet.

3. Show your work. Correct answers without enough work may receive no credit.

4. Find the required table(s) at the end of the packet. You may tear the tables sheet(s) free.

5. If a question is ambiguous, resolve the ambiguity in writing. We will consider grading accordingly.

6. The exam ends when I call time. If you continue writing after I call time, you risk a penalty. (The alternative, that you get more time than your peers, is unfair.)

7. You are welcome to turn your exam in to me before I call time. However, if you are still here in the last five minutes, please remain seated until I've called time.

Question	Points	Deduction
Q0 (cover)	1	
Q1	18	
Q2	6	
Q3	12	
Q4	18	
Q5	15	
Q6	9	
Q7	12	
Q8	9	
Total	100	

1

1. Notice below the labels A through E in the boxplot, and the label F in the histogram of an R data vector x. For each line of R code, below, write the label (one of A through F) whose x-coordinate is the output of that line.

_____ q = quantile(x); q[5]

_____ q = quantile(x); q[1]

_____ mean(x)

_____ median(x)

_____ q = quantile(x); q[2]

_____ q = quantile(x); q[4]

ANSWER:

<u>E</u> q = quantile(x); q[5]

<u>A</u> q = quantile(x); q[1]

<u>F</u> mean(x)

<u>C</u> median(x)

<u>B</u> q = quantile(x); q[2]

<u>D</u> q = quantile(x); q[4]
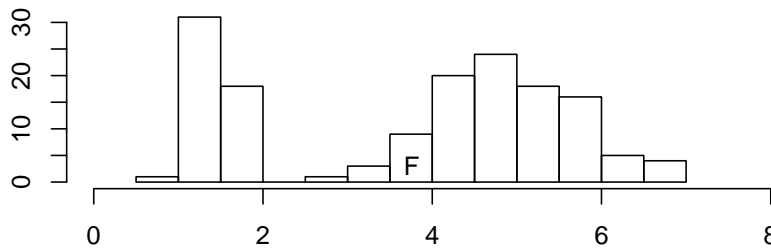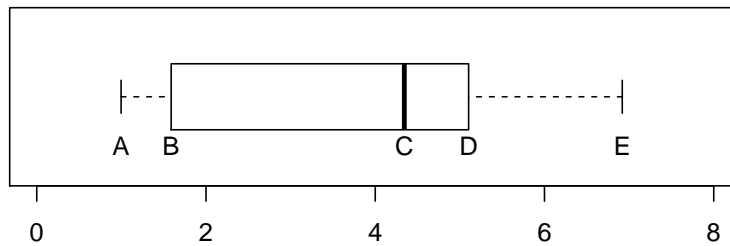
Note: F is the balance point of the data.

2. Consider this set of 10 spruce log lengths (in feet):

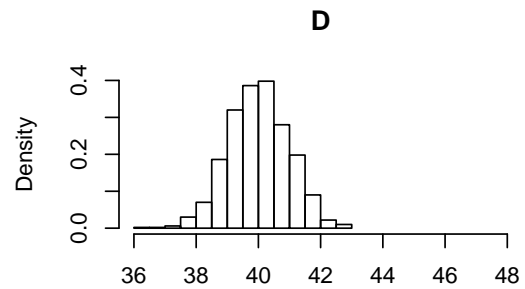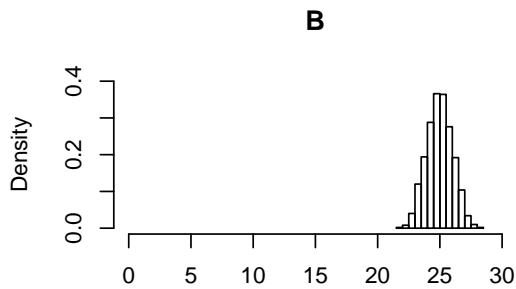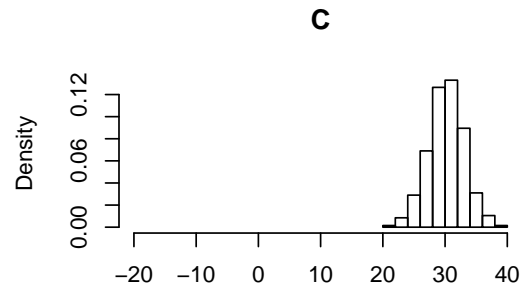9, 4, 4, 6, 9, 3, 9, 9, 7, 7

(a) Find the median, M. _____

(b) Find the first quartile, Q_1. _____

3. The histograms **A**, **B**, **C**, and **D**, summarize samples from four different populations.
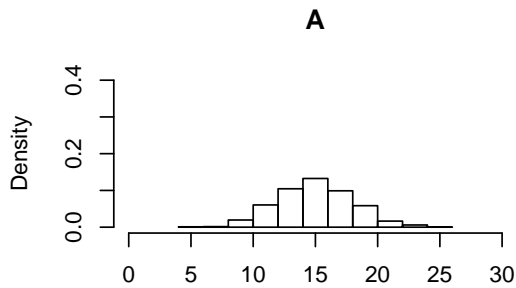
(a) For which sample, **A** or **B**, is the mean larger? _____

(b) For which sample, **A** or **B**, is the standard deviation larger? _____

(c) For which sample, **C** or **D**, is the mean larger? _____

(d) For which sample, **C** or **D**, is the standard deviation larger? _____

4. Consider the following R code and its output on data stored in an R vector, x:

```
> quantile(x)
   0%   25%   50%   75% 100%
 1.00 1.60 4.35 5.10 6.90
```

For each of the following summary statistics calculated from x, write its value or "NA" if there is not enough information given to find its value:

(a) $\bar{X}$ = sample mean: _____   ANSWER: NA

(b) M = sample median: _____   ANSWER: 4.35

(c) Q_1 = first quartile: _____   ANSWER: 1.60

(d) range: _____   ANSWER: $5.90 = 6.90 - 1.00$

(e) IQR = interquartile range: _____   ANSWER: $3.50 = 5.10 - 1.60$

(f) S = sample standard deviation: _____   ANSWER: NA

5. A simple random sample of 40 mosquitos is collected from a bog in Nunavut, Canada, and each bug is weighed. An approximate 95% percent confidence interval for the population mean weight, μ, of mosquitos in the bog is calculated from the sample as $\bar{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = (6.1, 7.0)$ mg. Mark each correct statement with "X".

   (a) _____ About 95 percent of the bog's mosquitos weigh between 6.1 and 7.0.
   ANSWER: No.

   (b) _____ About 95 percent of the sample's mosquitos weigh between 6.1 and 7.0.
   ANSWER: No.

   (c) _____ In repeated sampling, the method produces intervals that include the population mean approximately 95 percent of the time.
   ANSWER: Yes.

   (d) _____ In repeated sampling, the method produces intervals that include the sample mean approximately 95 percent of the time.
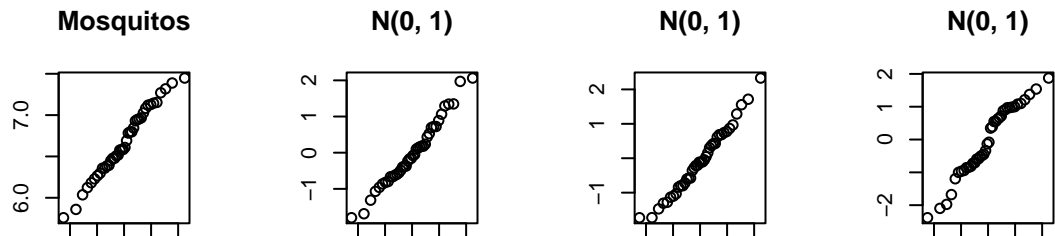   ANSWER: No.

   (e) _____ There is a probability of 0.95 that the population mean is between 6.1 and 7.0.
   ANSWER: No.

6. Another simple random sample of 40 mosquitos is collected and weighed.

   (a) Here is a normal QQ plot, on the left, of the sample. The other three QQ plots are of random samples of size 40 from the normal population $N(0, 1)$.

   Is it plausible the population of mosquito weights is normally distributed? (Mark your choice with "X".)   ____ Yes   or   ____ No
   ANSWER:
   Yes, as the mosquito QQ plot is reasonbly straight relative to the plots of data from $N(0, 1)$.

   (b) Here are summary statistics for the sample:

sample size n	sample mean $\bar{x}$	median M	range	sample standard deviation s
40	6.671	6.596	1.689	.422

   Suppose the population standard deviation σ is .500. Find a 97% confidence interval, expressed as "center $\pm$ error margin," for the population mean weight.

   ANSWER:
   $1 - \alpha = .97 \implies \alpha = .03$ and $z_{.03/2} = z_{.015} = 2.17$
   $\bar{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 6.671 \pm 2.17\frac{.500}{\sqrt{40}} = 6.671 \pm .172.$

(c) Suppose the researchers would like to make a 97% confidence interval whose error margin is 0.100. What sample size is required?

7. Jane has a biology class in Genetics Biotechnology Center which ends at 3:45. She has a Chemistry class at 4:00 in Chamberlin Hall. Her travel time between classes follows a normal distribution with mean 10 minutes and standard deviation 5 minutes.

   (a) If Jane leaves biology at 3:45, find the probability she will be late to chemistry.

   ANSWER:
   Let X be her travel time. Then $X \sim N\left(10, 5^2\right)$.
   $P(X > 15) = P\left(\frac{X-\mu}{\sigma} > \frac{15-10}{5}\right) = P(Z > 1) = P(Z < -1) = .1587$.

   (b) If Jane leaves biology at 3:45 each of the next four class days, find the probability her average travel time over those days will be less than 10 minutes.

   ANSWER:
   Let X_1, X_2, X_3, and X_4 be her travel times for the next 4 class days. Since $X_i \sim N\left(10, 5^2\right)$, $\bar{X} \sim N\left(10, \frac{5^2}{4}\right)$.
   $P(\bar{X} < 10) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < \frac{10-10}{5/\sqrt{4}}\right) = P(Z < 0) = .5$.

   (c) Tomorrow Jane has to arrive at chemistry before 4:00 for an exam. Find how at many minutes before 4:00 she should leave biology if she wants the probability she is on time to be 0.985.

   ANSWER:
   Let x be the number of minutes before 4:00 she should leave. Then $P(X \le x) = 0.985$, so $P\left(\frac{X-\mu}{\sigma} < \frac{x-10}{5}\right) = P(Z < z) = 0.985$. The required z with left-tail area .985 is $z = 2.17$, so $x = 2.17(5) + 10 = 20.85$.

   (d) Each day, Jane forgets her chemistry lab book with probability $\frac{1}{10}$, independently of what happened other days. Find the probability she forgets her lab book for exactly one of the 15 lab meetings during the semester.

   ANSWER:
   Let $X =$ number of labs of 15 for which she forgets her lab book. Then $X \sim \text{Bin}\left(n = 15, \pi = \frac{1}{10}\right)$.
   $P(X = 1) = \binom{15}{1}\left(\frac{1}{10}\right)^1\left(1 - \frac{1}{10}\right)^{15-1} = 15\left(\frac{1}{10}\right)\left(\frac{9}{10}\right)^{14} \approx .3432$

8. You are searching for an apartment in a new neighborhood and plan to ask three independent real estate agencies to estimate μ, the average rent in this neighborhood. Let $X_1, X_2,$ and X_3 be the average rent according to these agencies.

You know each of these agencies tends to inflate the actual average rental price by $100, on average. Specifically:
$$E(X_i) = \mu + 100 \quad \text{for } i = 1, 2, 3$$

In addition, agency 1 provides more reliable information than the others. Specifically,
$$VAR(X_1) = 100^2$$

and
$$VAR(X_2) = VAR(X_3) = 200^2$$

Consider the sample mean estimator for μ defined by $\hat{\mu} = \bar{X} = \frac{1}{3}\sum_{i=1}^{3} X_i$.

(a) Find the expected value of $\bar{X}$.
$E(\bar{X}) =$

ANSWER:

$$E(\bar{X}) = E\left(\frac{1}{3}\sum_{i=1}^{3} X_i\right)$$
$$= \frac{1}{3}E\left(\sum_{i=1}^{3} X_i\right)$$
$$= \frac{1}{3}\sum_{i=1}^{3} E(X_i)$$
$$= \frac{1}{3}\sum_{i=1}^{3} (\mu + 100)$$
$$= \mu + 100$$

(b) Mark each estimator for μ which is an improvement over $\bar{X}$. (Mark with "X".)

i. _____ $\hat{\mu} = \bar{X} - 100$ ANSWER: Yes.
ii. _____ $\hat{\mu} = \bar{X} - 50$ ANSWER: Yes.
iii. _____ $\hat{\mu} = \bar{X} + 50$ ANSWER: No.
iv. _____ $\hat{\mu} = \bar{X} + 100$ ANSWER: No.

ANSWER:
On average $\bar{X}$ is $100 too high, $\bar{X} - 50$ is 50 too high, and $\bar{X} - 100$ is correct. Thus the latter two estimators are better than $\bar{X}$. (Note that all these estimators have the same variance.)
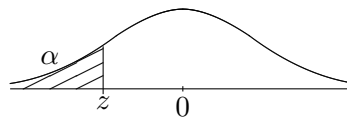
8

(c) Find the variance of $\bar{X}$.

$$VAR(\bar{X}) =$$

$$
\begin{aligned}
VAR(\bar{X}) &= VAR\left(\frac{1}{3}\sum_{i=1}^{3} X_i\right) \\
&= \frac{1}{3^2} VAR\left(\sum_{i=1}^{3} X_i\right) \\
&= \frac{1}{9}\sum_{i=1}^{3} VAR(X_i) \text{ (by independence)} \\
&= \frac{1}{9}\left(100^2 + 200^2 + 200^2\right) \\
&= 10000 \\
&= 100^2
\end{aligned}
$$

For $z = a.bc$, look in row $a.b$ and column $.0c$ to find $P(Z < z)$. e.g.
For $z = 1.42$, look in row 1.4 and column .02 to find $P(Z < 1.42) = .9222$ (on next page).

Cumulative $N(0,1)$ Distribution, $z \leq 0$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
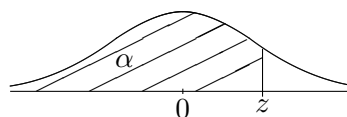
Cumulative $N(0,1)$ Distribution, $z \geq 0$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999