1. A random sample of 35 measurements of wolf jaw lengths have mean $\bar{x} = 10.3229$ cm and standard deviation $s = .3317$.

(a) Find a 95% confidence interval for the true mean length. Suppose that the measurements are normally distributed.

ANSWER:

$$10.3229 \pm t_{(35-1, .025)} \frac{.3317}{\sqrt{35}} = 10.3229 \pm 2.03(.0561) = 10.3229 \pm .1138$$

(Oops: this CI is from §5 before exam 1, so this question is not suitable for exam 2.)

(b) Are the wolf jaw length data in the previous problem strong evidence that the population mean length is different than 10 cm? Use a hypothesis test to answer.

ANSWER:

• Hypotheses: $H_0 : \mu = 10$ vs. $H_A : \mu \neq 10$
• Assumptions: We have a SRS. Since $(n = 35) > 30$, I’ll use our rule-of-thumb and suppose that the CLT applies, so $\bar{X}$ is approximately normal and I’ll use a one-sample t-test.
• Test statistic: $t = \frac{10.3229 - 10}{.3317/\sqrt{35}} = 5.76$
• P-value: $P(T_{34} < -5.76) + P(T_{34} > 5.76) = 2P(T_{34} > 5.76) < 2(.001) = .002$ (I looked in table rows 30 and 40.)
• Conclusion: Reject $H_0$. The data are strong evidence that the population mean jaw length is different than 10 cm.

2. In a study of U.S. currency, heroin was found on 7 of a random sample of 50 bills.

(a) Find a 95% confidence interval for the true proportion of bills tainted with heroin.

ANSWER:

Note that the observed numbers of successes and failures, 7 and 43 = 50 − 7, are both > 5. $\hat{p} = \frac{7}{50} = .14$; margin = $z_{.025} \sqrt{\frac{.14(1-.14)}{50}} = 1.96(.04907) = .096$; interval = $.14 \pm .096$

(b) Test whether the population proportion of heroin-tainted bills is different than .12.

ANSWER:

• Hypotheses: $H_0 : \pi = .12$ vs. $H_A : \pi \neq .12$
• Assumptions: We have a SRS. $n = 50$ and $\pi = .12 \implies$ the expected numbers of successes and failures are $6 = 50(.12)$ and $44 = 50(1 -.12)$, which are both > 5. I’ll use the test for one proportion.
• Test statistic: $p = \hat{p} = \frac{7}{50} = .14 \implies z = \frac{.14 - .12}{\sqrt{.12(1-.12) / 50}} \approx 0.44$
• P-value: $P(Z < -0.44) + P(Z > 0.44) = 2P(Z < -0.44) = 2(.3300) = .66$
• Conclusion: Do not reject $H_0$. The data are not strong evidence that the population proportion of tainted bills is different than .12.
3. (a) What effect does reducing the value of the significance level of a test from $\alpha = .05$ to $\alpha = .01$ have on the probability of committing a type I error?

**ANSWER:**
It decreases it, as the significance level and the probability of a type I error are the same number.

(b) What effect does reducing $\beta$, the probability of making a type II error in a test, have on the test’s power?

**ANSWER:** It increases power, since power = $1 - \beta$.

4. Here are body temperature and brain temperature (in °C) for six ostriches:

<table>
<thead>
<tr>
<th>Ostrich</th>
<th>Body temperature</th>
<th>Brain temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>38.51</td>
<td>39.32</td>
</tr>
<tr>
<td>2</td>
<td>38.45</td>
<td>39.21</td>
</tr>
<tr>
<td>3</td>
<td>38.27</td>
<td>39.20</td>
</tr>
<tr>
<td>4</td>
<td>38.52</td>
<td>38.68</td>
</tr>
<tr>
<td>5</td>
<td>38.62</td>
<td>39.09</td>
</tr>
<tr>
<td>6</td>
<td>38.18</td>
<td>38.94</td>
</tr>
</tbody>
</table>

(a) To check assumptions, what graph(s) would you like to make and why? (Don’t make any graphs.)

**ANSWER:**
These samples are paired by bird. I would make a QQ plot of the six differences within each pair to check whether it is reasonable to assume the population of differences is normally distributed.

(b) Suppose, from your graph(s), it is reasonable to assume normality. Test whether the true mean body temperature is different than the true mean brain temperature.

**ANSWER:**
I’ll use a paired $t$-test.

- Hypotheses: $H_0 : \mu_d = 0$ vs. $H_A : \mu_d \neq 0$
- Assumptions: We have a SRS of birds. The population of differences is normal.
- Test statistic: $\bar{d} = .6483$ and $s_d = .2831$, so $t = \frac{.6483}{.2831/\sqrt{6}} = 5.61$
- P-value: $P(T_{6-1} < -5.61) + P(T_{6-1} > 5.61) = 2P(T_5 > 5.61) = 2(\text{between } .001 \text{ and } .005) = \text{between } .002 \text{ and } .010$
- Conclusion: Reject $H_0$. The data are strong evidence that, in the population of birds, the mean body temperature is different than the mean brain temperature.

5. In the early 1950s, a randomly-selected 200745 children were given Jonas Sauk’s new vaccine for polio, and another 201229 were given a placebo. 82 in the vaccine group got polio, while 162 in the placebo group got polio. Is it reasonable to conclude that the population proportion of vaccinated children who get polio is smaller than the population proportion of children who receive a placebo?

**ANSWER:**
Hypotheses: $H_0 : \pi_V - \pi_P = 0$, $H_1 : \pi_V - \pi_P < 0$, where $\pi_V =$ population proportion of vaccinated children who get polio and $\pi_P =$ population proportion of children who receive a placebo children who get polio.

Assumptions: The data were sampled independently from $\text{Bin}(n = 200745, \pi = \pi_V)$ and $\text{Bin}(n = 201229, \pi = \pi_P)$. The observed numbers of successes and failures in each sample are all way bigger than 5, so we’ll use the test for the difference between two proportions.

Test statistic: $z = \frac{\frac{82}{200745} - \frac{162}{201229} - 0}{\sqrt{\frac{82}{200745} + \frac{162}{201229} \left( 1 - \frac{82+162}{200745+201229} \right)}} \approx -5.104$

P-value: $P(Z < -5.104) \approx 0$ (The table says it’s < .0001; software says it’s about $1.66 \times 10^{-24}$.)

Conclusion: Reject $H_0$. The data are extremely strong evidence that the population proportion of children who receive the vaccine and get polio is smaller than the proportion of children on a placebo who get polio.