STAT 371 Exam 2 NetID (mine is "jgillett" from "jgillett@wisc.edu"): ____________

Last name: ______________________________ First name: ________________________

Discussion (check one):

____ 321 Tu 12:05-12:55 Engineering Hall 2239 Fan, Ning and Yu, Zhongjie

____ 322 Tu 4:35-5:25 Microbial Sciences 1420 Hu, Bowen and Lin, Yunong

____ 323 We 8:50-9:50 Engineering Hall 3032 Fan, Ning and Lin, Yunong

Instructions.

1. Do not open the exam until I say "go."
2. Put away everything except a pencil, a calculator, and your two pages (two sides) of notes.
3. Show your work clearly. Correct answers without enough work may receive no credit.
4. Find the needed table(s) at the end of the packet. You may tear the tables sheet(s) free.
5. If a question is ambiguous, resolve it in writing. We will consider grading accordingly.
6. The exam will end when I call time. If you continue writing after I call time, you risk a penalty. (The alternative, that you get more time than your peers, is unfair.)
7. You are welcome to turn your exam in to me before I call time. However, if you are still here in the last five minutes, please remain seated until I've picked up all the exams.
8. Good luck!

Question	Points	Your Score
Q1	10	
Q2	10	
Q3	20	
Q4	20	
Q5	10	
Q6	10	
Q7	20	
Total	100	

1. The Wisconsin State Patrol is worried its fleet of vehicles–which includes 500 police cruisers, motorcycles, and SUVs–is aging. It takes a simple random sample of 35 of its vehicles and finds 10 have traveled more than 100,000 miles. Find a 90% confidence interval for π, the unknown proportion of fleet vehicles which have traveled over 100,000 miles, expressed as "center $\pm$ error margin."

2. Consider a Z test of $H_0 : \mu = \mu_0$ vs. $H_A : \mu < \mu_0$ where σ is known. Mark each statement TRUE or FALSE.

   (a) __________ The rejection region corresponding to significance level $\alpha = .05$ contains the rejection region corresponding to $\alpha = .10$.

   (b) __________ We cannot make both a type I error and a type II error at the same time.

   (c) __________ If $\bar{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ contains μ_0, then we reject H_0 at level α.

   (d) __________ The p-value is the probability H_0 is true.

   (e) __________ The p-value is the probability H_A is true.

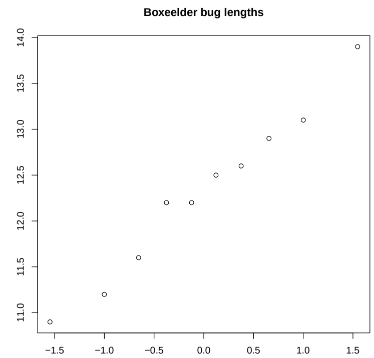
3. A neuroscientist gathered a sample of 80 *Drosophila Melanogaster*(fruitflies) and found that 55 reacted when prodded with a needle heated at $41°C$. If more than 62% of the flies react, she will need to recalibrate the heated stimulus.

   (a) What hypotheses should she test to find out whether recalibration is necessary?

   (b) Choose an appropriate test and find the p-value.

   (c) Make decision at the $\alpha = .05$-level in the context of the problem.

4. The sunny south wall of a house was covered with boxelder bugs. A researcher enclosed the wall in plastic to capture all the bugs. The lengths of a simple random sample of 10 bugs were measured in mm: 10.9, 11.2, 11.6, 12.2, 12.2, 12.5, 12.6, 12.9, 13.1, 13.9. For these data, $\bar{x} \approx 12.31$ and $s \approx 0.90$. Here is a QQ plot:



(a) Is it plausible that the population of lengths is normally distributed? Why or why not?

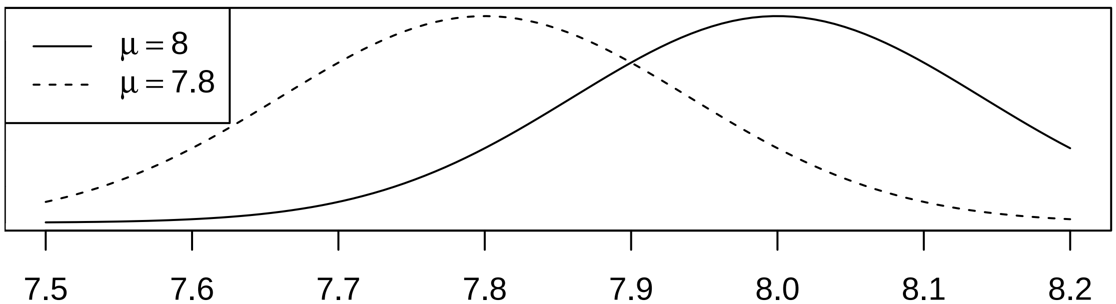
(b) Suppose the population of lengths is normal. Find a 95% confidence interval, expressed as "center $\pm$ error margin," for the unknown population mean length.

(c) Find the test statistic and p-value for a test of $H_0 : M = 11$ vs. $H_A : M > 11$, where M is the population median length. Draw a conclusion using significance level 0.05. (Here are the data again: 10.9, 11.2, 11.6, 12.2, 12.2, 12.5, 12.6, 12.9, 13.1, 13.9.)

5. A rubber band maker wants the unstretched lengths of their bands centered at 8.0 centimeters. They run a test in which they will reject $H_0 : \mu = 8.0$, where μ is the population mean length, if a simple random sample of size 100 has mean below 7.9 or above 8.1.

   Suppose the true distribution of lengths is $N(7.8, 1.4^2)$.

   (a) Complete the drawing of two distributions of $\bar{X}$, below, by marking the rejection region and shading the area corresponding to the power of the test.



   (b) Find the power of the test for $\mu_A = 7.8$.

6. Here is a summary of how we used bootstrapping to form a confidence interval and run a hypothesis test for an unknown mean, μ:

- Draw simple random sample of size n from the population. Find $\bar{x}$ and s.
- Resample $x_1^*, \ldots, x_n^*$ with replacement from data. Find $\bar{x}^*$, s^* and $\hat{t} = \dfrac{\bar{x}^* - \bar{x}}{s^*/\sqrt{n}}$.
- Repeat previous step B times to get many $\hat{t}$s.
- For a confidence interval, find $1 - \frac{\alpha}{2}$ and $\frac{\alpha}{2}$ upper critical values $\hat{t}_{(1-\alpha/2)}$ and $\hat{t}_{(\alpha/2)}$; $\left(\bar{x} - \hat{t}_{(\alpha/2)} \frac{s}{\sqrt{n}}, \bar{x} - \hat{t}_{(1-\alpha/2)} \frac{s}{\sqrt{n}}\right)$ contains μ for about $100(1-\alpha)\%$ of original samples
- To test $H_0 : \mu = \mu_0$, find $t_{\text{obs}} = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$ and use p-value $= \frac{m}{B}$, where
  $H_A : \mu > \mu_0 \implies m = \#\hat{t}$ with $\hat{t} > t_{\text{obs}}$
  $H_A : \mu < \mu_0 \implies m = \#\hat{t}$ with $\hat{t} < t_{\text{obs}}$
  $H_A : \mu \neq \mu_0 \implies m = \#\hat{t}$ with $|\hat{t}| > |t_{\text{obs}}|$

Mark each statement about bootstrapping TRUE or FALSE.

(a) _________ Increasing n decreases the variability in $\bar{X}$.
(Here (capital) $\bar{X}$ is the random variable before sampling, and (lower case) $\bar{x}$ is the number after sampling. I'm not trying to trick anybody on this distinction.)

(b) _________ Increasing B (while leaving n fixed) decreases the variability in $\bar{X}$.

(c) _________ Increasing n (while leaving B fixed) makes the resampled distribution of $\hat{t}$ values more like the distribution of t values in the population.

(d) _________ Increasing B (while leaving n and the original sample fixed) decreases the variability in the interval from one bootstrap interval computation to the next.

(e) _________ As B increases (and n is fixed), the distribution of $\hat{t}$ values approaches $N(0,1)$.

7. A coconut grower tested the yields of tall and dwarf coconut trees by weighing the fruits of simple random samples of 154 coconuts from his tall trees and 40 coconuts from his dwarf trees. Here are the resulting weights in ounces:

Cocnut variety	sample mean	sample standard deviation	sample size n
Tall	24.28	2.63	154
Dwarf	21.99	2.71	40

QQ plots are compatible with normal populations. Are these data strong evidence, at significance level 0.05, that coconuts from Tall trees are heavier than those from Dwarf trees?

- hypotheses:

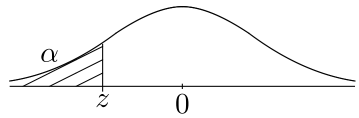
- assumptions:

- test statistic:

- p-value:

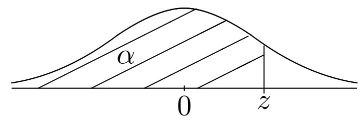
- conclusion: yes / no (circle one)

For $z = a.bc$, look in row $a.b$ and column $.0c$ to find $P(Z < z)$. e.g.
For $z = 1.42$, look in row 1.4 and column .02 to find $P(Z < 1.42) = .9222$ (on next page).



Cumulative $N(0,1)$ Distribution, $z \leq 0$

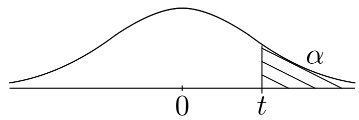
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641



Cumulative $N(0,1)$ Distribution, $z \geq 0$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	$\boxed{.9222}$	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999

Upper Tail Points for the Student's t Distributions



$\nu = n-1$	.25	.10	.05	.025	.01	.005	.001
1	1.000	3.078	6.314	12.706	31.821	63.657	318.309
2	.816	1.886	2.920	4.303	6.965	9.925	22.327
3	.765	1.638	2.353	3.182	4.541	5.841	10.215
4	.741	1.533	2.132	2.776	3.747	4.604	7.173
5	.727	1.476	2.015	2.571	3.365	4.032	5.893
6	.718	1.440	1.943	2.447	3.143	3.707	5.208
7	.711	1.415	1.895	2.365	2.998	3.499	4.785
8	.706	1.397	1.860	2.306	2.896	3.355	4.501
9	.703	1.383	1.833	2.262	2.821	3.250	4.297
10	.700	1.372	1.812	2.228	2.764	3.169	4.144
11	.697	1.363	1.796	2.201	2.718	3.106	4.025
12	.695	1.356	1.782	2.179	2.681	3.055	3.930
13	.694	1.350	1.771	2.160	2.650	3.012	3.852
14	.692	1.345	1.761	2.145	2.624	2.977	3.787
15	.691	1.341	1.753	2.131	2.602	2.947	3.733
16	.690	1.337	1.746	2.120	2.583	2.921	3.686
17	.689	1.333	1.740	2.110	2.567	2.898	3.646
18	.688	1.330	1.734	2.101	2.552	2.878	3.610
19	.688	1.328	1.729	2.093	2.539	2.861	3.579
20	.687	1.325	1.725	2.086	2.528	2.845	3.552
21	.686	1.323	1.721	2.080	2.518	2.831	3.527
22	.686	1.321	1.717	2.074	2.508	2.819	3.505
23	.685	1.319	1.714	2.069	2.500	2.807	3.485
24	.685	1.318	1.711	2.064	2.492	2.797	3.467
25	.684	1.316	1.708	2.060	2.485	2.787	3.450
26	.684	1.315	1.706	2.056	2.479	2.779	3.435
27	.684	1.314	1.703	2.052	2.473	2.771	3.421
28	.683	1.313	1.701	2.048	2.467	2.763	3.408
29	.683	1.311	1.699	2.045	2.462	2.756	3.396
30	.683	1.310	1.697	2.042	2.457	2.750	3.385
40	.681	1.303	1.684	2.021	2.423	2.704	3.307
60	.679	1.296	1.671	2.000	2.390	2.660	3.232
100	.677	1.290	1.660	1.984	2.364	2.626	3.174
∞	.674	1.282	1.645	1.960	2.326	2.576	3.090

The column heads .25 to .001 are values of α.