STAT 371 Exam 2 NetID (mine is “jgillett” from “jgillett@wisc.edu”): ____________

Last name: ______________________________ First name: ________________________

Discussion (check one):

- Lecture 371-001 TuTh 9:30-10:45 Microbial Sciences 1520:

  ____ 311 Tu 1:20-2:10 Engineering Hall 2317 Kent, Sean and Mu, Yuqin

  ____ 312 Tu 2:25-3:15 Mechanical Engineering 1153 Kent, Sean and Mu, Yuqin

  ____ 313 We 11:00-11:50 Van Hise 594 Xu, Yuqing and Bobst, Claire

- Lecture 371-002 TuTh 11:00-12:15 Ingraham Hall 19:

  ____ 321 We 12:05-12:55 Van Hise 594 Xu, Yuqing and Bobst, Claire

  ____ 322 Tu 2:25-3:15 Nutritional Sciences 290 Zhang, Chengning and Huang, Kunling

  ____ 323 We 8:50-9:40 Van Hise 494 Zhang, Chengning and Huang, Kunling

Instructions:

1. Do not open the exam until I say “go.”
2. Put away everything except a pencil, a calculator, and your two one-page (two sides each) notes sheets.
3. Show your work. Correct answers without enough work may receive no credit.
4. Find the required table(s) at the end of the packet. You may tear the tables sheet(s) free.
5. If a question is ambiguous, resolve the ambiguity in writing. We will consider grading accordingly.
6. The exam ends when I call time. If you continue writing after I call time, you risk a penalty. (The alternative, that you get more time than your peers, is unfair.)
7. You are welcome to turn your exam in to me before I call time. However, if you are still here in the last five minutes, please remain seated until I've called time.

Question	Points	Deduction
Q1	10	
Q2	20	
Q3	20	
Q4	20	
Q5	10	
Q6	20	
Total	100	

1. Consider a hypothesis test. Circle the best answer after each question.

   (a) What effect does increasing the significance level of a test from .05 to .10 have on the probability of committing a type I error?

      i. It decreases it.
      ii. It increases it.

   ANSWER:

   (ii) It increases it, as the significance level and the probability of a type I error are the same number (called α).

   (b) What effect does increasing power have on a test's probability of making a type II error?

      i. It decreases it.
      ii. It increases it.

   ANSWER:

   (i) It decreases it, since power $= 1 - \beta$, and β is the probability of a type II error.

   (c) What effect does decreasing the probability of a type I error have on power?

      i. It decreases it.
      ii. It increases it.

   ANSWER:

   (i) It decreases it, since power $=$ P(reject $H_0|H_0$ is false), and decreasing the probability of a type I error makes it less likely that H_0 is rejected.

   (d) Suppose you wish to do inference about an unknown population mean, μ, from a simple random sample of size 10, and you know the population standard deviation, σ. In this setting, it is appropriate to use a Z confidence interval and a Z two-sided test.

   Instead, you use a t confidence interval and two-sided test, even though you know σ.

      i. What does using t instead of Z do to the error margin of your confidence interval?

         A. It decreases it.
         B. It increases it.

      ANSWER:

      (B) It increases it because $t_{10-1,\alpha/2} > z_{\alpha/2}$ for all $0 < \alpha < 1$, as you can see by looking down any column of the t table to its last line, the one for the t distribution with ∞ degrees of freedom, which is the same as the Z distribution.

      ii. What does using t instead of Z do to the p-value of your test?

         A. It decreases it.
         B. It increases it.

      ANSWER:

      (B) It increases it because the t_{10-1} distribution is shorter with thicker tails than the Z distribution, so for any given test statistic, the two tails of the t_{10-1} distribution have greater area than the two tails of the Z distribution.

2. A simple random sample of quarter-pound hamburger patties was weighed, with these results (in ounces):

   3.9 4.4 4.2 3.6 3.9 4.1 3.9

   Summary statistics include $n = 7$, $\bar{x} = 4$, and $s = .26$. 10000 resamples with replacement are taken for a bootstrap, with a $\hat{t}$ calculated for each resample. Here are some quantiles of the resampling distribution of $\hat{t}$:

probability	.001	.005	.01	.05	.10	.50	.90	.95	.99	.995	.999
quantile	-11.1	-3.8	-3.4	-2.1	-1.5	0.0	1.4	1.9	3.1	3.5	5.7

   (a) The bootstrap distribution of $\hat{t}$ values approximates what distribution? Circle the best answer.

   i. $N(0, 1)$
   ii. t_{n-1}
   iii. The distribution of $T = \frac{\bar{X}-4}{s/\sqrt{7}}$ in the population of patties.

   ANSWER:

   (iii)

   (b) Find a 99% bootstrap confidence interval for the population mean weight of quarter-pound hamburger patties.

   ANSWER:

   For a 99% interval, we have $1 - \alpha = .99 \implies \alpha = .01 \implies \alpha/2 = .005$, and we need the .005 and .995 upper critcal values, $\hat{t}_{.005} = 3.5$ and $\hat{t}_{.995} = -3.8$. Here is the interval: $\left(\bar{x} - \hat{t}_{(\alpha/2)}\frac{s}{\sqrt{n}}, \bar{x} - \hat{t}_{(1-\alpha/2)}\frac{s}{\sqrt{n}}\right) = \left(4 - (3.5)\frac{.26}{\sqrt{7}}, 4 - (-3.8)\frac{.26}{\sqrt{7}}\right) = (3.66, 4.37)$

   (c) Estimate the p-value for a bootstrap test of $H_0 : \mu = 4$ vs. $H_A : \mu > 4$, where μ is the population mean weight. (Hint: You cannot use R to estimate the p-value as we did in class. You can, nevertheless, estimate the p-value and draw a conclusion.)

   ANSWER:

$$
\begin{aligned}
p\text{-value} &= P(T > t_{obs}) \\
&= P(T > \frac{4-4}{.26/\sqrt{7}}) \\
&= P(T > 0) \\
&\approx .5 \text{ (because the .50 quantile is 0)}
\end{aligned}
$$

3. In a study of self-medication, a simple random sample of 1230 adults completed a survey. The survey reported that 441 of the adults had a cough during the past month.

   (a) What hypotheses should be tested to find out whether, in the population from which the sample was drawn, less than half of the adults had a cough during the past month?

   ANSWER:
   $H_0 : \pi = 0.5$ vs. $H_A : \pi < 0.5$

   (b) Choose an appropriate test, check assumptions, and find the p-value.

   ANSWER:
   First we check that the expected numbers of successes and failures are each greater than 5:
   $n\pi_0 = 1230(.5) = 615 > 5$ and $n(1 - \pi_0) = 1230(1 - .5) = 615 > 5$
   We have $x = 441$ successes in $n = 1230$ trials, for a sample proportion $p = \frac{441}{1230} = \frac{11}{16} = .3585$.
   Our test statistic is $Z = \dfrac{.3585 - .5}{\sqrt{.5(1 - .5)/1230}} \approx -9.9$ (Using $p = \frac{441}{1230}$ for π_0 is a small error that yields $Z \approx -10.3$.)
   The p-value is $P(Z < -9.9) < .0001$ from the table (or 2.08×10^{-23} from software).

   (c) Make decision at the $\alpha = .05$-level in the context of the problem.

   ANSWER:
   Reject H_0. The data are strong evidence that $\pi < .5$ and less than half of the population had a cough in the past month.

   (d) What is the smallest sample size we could have used for this test while still meeting the assumptions it requires?

   ANSWER:
   We need $n\pi_0 > 5$ and $n(1-\pi_0) > 5$, that is $n(.5) > 5$ and $n(1-.5) > 5$, which is $n > 10$. So $n = 11$ is the smallest sample size we could have used.

4. A simple random sample of 5 (young) spruce trees from a nursery weighed 132, 145, 162, 166, and 175 grams. A simple random sample of 4 fir trees from the same nursery weighed 137, 147, 158, and 170 grams.

   (a) Suppose each species' population of weights is normally distributed. Decide whether the population mean weights of spruce and fir in this nursery are different.

   - Hypotheses:
   - Assumptions:
   - Test statistic:
   - P-value:
   - Conclusion:

   ANSWER:

   - Hypotheses: $H_0 : \mu_{spruce} - \mu_{fir} = 0$, $H_A : \mu_{spruce} - \mu_{fir} \neq 0$, where μ_{spruce} = the population mean spruce weight and μ_{fir} = the population mean fir weight.
   - Assumptions: We have two independent SRSs from two normal populations. Since the two sample variances, 298.5 and 202, are within a factor of 2, we assume equal variances and proceed with a 2-sample t-test.
   - Test statistic: $\overline{spruce} = 156, \overline{fir} = 153, s_{spruce} = 17.277, s_{fir} = 14.213, s^2_{\text{pooled}} = \sqrt{\frac{(5-1)298.5+(4-1)202}{5+4-2}} = 257.1 \implies t = \frac{(156-153)-0}{\sqrt{257.1/5+257.1/4}} = 0.279.$
   - P-value: $P(T_{5+4-2} < -0.279) + P(T_7 > 0.279) = 2P(T_7 > 0.279) > 2(0.25) = 0.50$ (R gives 0.788).
   - Conclusion: Do not reject H_0. The data are not strong evidence that the nursery's spruce trees have a different population mean weight than its firs.

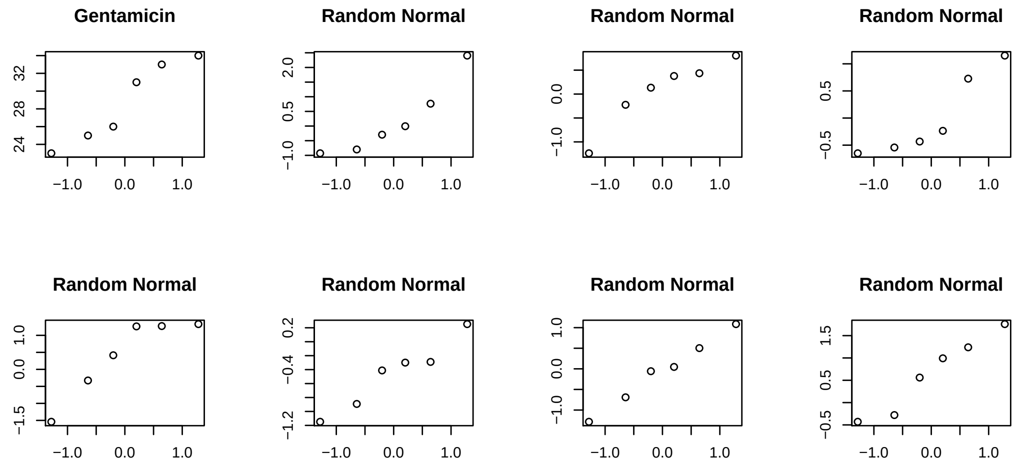
   (b) Suppose each species' population of weights is normally distributed. Find a 98% confidence interval for $\mu_{spruce} - \mu_{fir}$. (Hint: The problem says "98%," not "95%.")

   ANSWER:

   $t_{\nu,\alpha/2} = t_{5+4-2,.01} = t_{7,.01} = 2.998 \implies \text{interval} = (\overline{spruce} - \overline{fir}) \pm t_{n_{spruce}+n_{fir}-2,\alpha/2}\sqrt{s_p^2/n_{spruce} + s_p^2/n_{f}}$

   $\approx (156 - 153) \pm 2.998\sqrt{257.1/5 + 257.1/4} = 3 \pm 32.25 = (-29.25, 35.25)$

5. Six female Suffolk sheep were injected with the antibiotic Gentamicin, at a dosage of 10 mg/kg body weight. Their blood serum concentration (μg/ml) of Gentamicin 1.5 hours after injection were as follows: 33, 26, 34, 31, 23, 25. For these data, the sample mean is 28.7 and the sample standard deviation is 4.6. Here is a QQ plot of the Gentamicin data (top left), along with 7 QQ plots of random samples of size 6 from a normal population for reference:



(a) Is it plausible that the population of concentrations is normally distributed? Circle the best answer.

i. Yes, because the Gentamicin plot looks reasonably linear.
ii. No, because the Gentamicin plot looks nonlinear.
iii. Yes, because most of the 8 plots look reasonably linear.
iv. No, because most of the 8 plots look nonlinear.
v. Yes, because the departure from linear in the Gentamicin plot is no more than that of many of the 7 Random Normal plots.
vi. No, because the departure from linear in the Gentamicin plot is more than that of most of the 7 Random Normal plots.
vii. Yes, because $n = 6$ is large enough by our rule of thumb requiring $n > 5$.
viii. No, because $n = 6$ is too small by our rule of thumb requiring $n > 30$.

ANSWER:
(v) is best.

(b) Suppose the population is normally distributed. Find a 95% confidence interval for the unknown population mean.

ANSWER:
For 95% confidence we have $1-\alpha = .95 \implies \alpha = .05$ and we need $t_{n-1,\alpha/2} = t_{6-1,.05/2} = t_{5,.025} = 2.571$. Our interval is $\bar{X} \pm t_{n-1,\alpha/2}\frac{S}{\sqrt{n}} = 28.7 \pm 2.571\frac{4.6}{\sqrt{6}} = 28.7 \pm 4.8$.

6. An 11-year-old girl named Mary has her temperature taken at 6 randomly chosen times throughout a day, with these results (in °F):

   98.3, 101.4, 99.3, 99.9, 99.8, 100.6

   (a) Use a sign test to decide whether Mary has a fever on the day of these measurements. Suppose a fever is defined as having a median temperature above 98.6°F, and use significance level $\alpha = .05$.

   ANSWER:

   - Hypotheses: $H_0 : M = 98.6$ vs. $H_A : M > 98.6$
   - Assumptions: We have a SRS from the population of Mary's temperatures that day, which has median M.
   - Test statistic: B = number of positive differences from the median, which, under H_0, is $\sim Bin(n = 6, \pi = .5)$. For our data with 5 of 6 values greater than 98.6, we have $b = 5$.
   - P-value: $P(B \geq 5) = P(B = 5) + P(B = 6) = \binom{6}{5}.5^5(1 - .5)^{6-5} + \binom{6}{6}.5^6(1 - .5)^{6-6} = 6(.5^6) + 1(.5^6) \approx 0.11$.
   - Conclusion: Our p-value, .11, is not smaller than $\alpha = .05$, so do not reject H_0. The data are not strong evidence that Mary has a fever.

   (b) Circle the best answer to each of these questions:

      i. Based on your conclusion, which kind of error could you have made?

         A. type I error

         B. type II error

         ANSWER:

         B. We did not reject H_0, so we could have made a type II error.

      ii. In light of the data, what do you think of your conclusion?

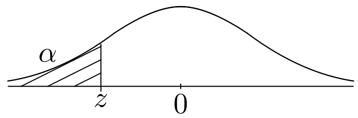
         A. My conclusion is correct.

         B. I made an error.

         ANSWER:

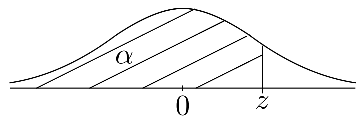
         B. I think I made a type II error by failing to reject H_0 when it is false. That is, I think Mary has a fever, but I worry that the sign test is not powerful enough to discern her fever. (Some other answers are also acceptable.)

For $z = a.bc$, look in row $a.b$ and column $.0c$ to find $P(Z < z)$. e.g.
For $z = 1.42$, look in row 1.4 and column .02 to find $P(Z < 1.42) = .9222$ (on next page).



Cumulative $N(0, 1^2)$ Distribution, $z \leq 0$

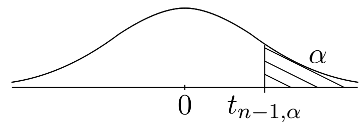
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641



Cumulative $N(0, 1^2)$ Distribution, $z \geq 0$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999

Upper Tail Points $t_{n-1,\alpha}$ for the Student's t_{n-1} Distributions



degrees of freedom	right-tail area α						
$n-1$	.25	.10	.05	.025	.01	.005	.001
1	1.000	3.078	6.314	12.706	31.821	63.657	318.309
2	.816	1.886	2.920	4.303	6.965	9.925	22.327
3	.765	1.638	2.353	3.182	4.541	5.841	10.215
4	.741	1.533	2.132	2.776	3.747	4.604	7.173
5	.727	1.476	2.015	2.571	3.365	4.032	5.893
6	.718	1.440	1.943	2.447	3.143	3.707	5.208
7	.711	1.415	1.895	2.365	2.998	3.499	4.785
8	.706	1.397	1.860	2.306	2.896	3.355	4.501
9	.703	1.383	1.833	2.262	2.821	3.250	4.297
10	.700	1.372	1.812	2.228	2.764	3.169	4.144
11	.697	1.363	1.796	2.201	2.718	3.106	4.025
12	.695	1.356	1.782	2.179	2.681	3.055	3.930
13	.694	1.350	1.771	2.160	2.650	3.012	3.852
14	.692	1.345	1.761	2.145	2.624	2.977	3.787
15	.691	1.341	1.753	2.131	2.602	2.947	3.733
16	.690	1.337	1.746	2.120	2.583	2.921	3.686
17	.689	1.333	1.740	2.110	2.567	2.898	3.646
18	.688	1.330	1.734	2.101	2.552	2.878	3.610
19	.688	1.328	1.729	2.093	2.539	2.861	3.579
20	.687	1.325	1.725	2.086	2.528	2.845	3.552
21	.686	1.323	1.721	2.080	2.518	2.831	3.527
22	.686	1.321	1.717	2.074	2.508	2.819	3.505
23	.685	1.319	1.714	2.069	2.500	2.807	3.485
24	.685	1.318	1.711	2.064	2.492	2.797	3.467
25	.684	1.316	1.708	2.060	2.485	2.787	3.450
26	.684	1.315	1.706	2.056	2.479	2.779	3.435
27	.684	1.314	1.703	2.052	2.473	2.771	3.421
28	.683	1.313	1.701	2.048	2.467	2.763	3.408
29	.683	1.311	1.699	2.045	2.462	2.756	3.396
30	.683	1.310	1.697	2.042	2.457	2.750	3.385
40	.681	1.303	1.684	2.021	2.423	2.704	3.307
60	.679	1.296	1.671	2.000	2.390	2.660	3.232
100	.677	1.290	1.660	1.984	2.364	2.626	3.174
∞	.674	1.282	1.645	1.960	2.326	2.576	3.090