STAT 371 Exam 2 NetID (mine is “jgillett” from “jgillett@wisc.edu”): ____________

Last name: ______________________________ First name: ________________________

Discussion (check one):

____ 311 TuTh 10:20-11:10 Mechanical Engineering 1152 Brown, Jared and Huang, Kunling

____ 312 TuTh 10:20-11:10 Russel Labs 104 Hu, Bowen and Cao, Wenzhi

____ 321 TuTh 1:10-2:00 Van Hise 387 Xu, Yuqing and Huang, Kunling

Instructions.

1. Do not open the exam until I say “go.”
2. Put away everything except a pencil, a calculator, and your two pages (two sides each) of notes (formula sheets).
3. Attempt all questions.
4. Show your work clearly. Correct answers without enough work may receive no credit.
5. Find the needed table(s) at the end of the packet. You may tear the tables sheet(s) free.
6. If a question is ambiguous, resolve it in writing. We will consider grading accordingly.
7. The exam will end when I call time. If you continue writing after I call time, you risk a penalty. (The alternative, that you get more time than your peers, is unfair.)
8. You are welcome to turn your exam in to me before I call time. However, if you are still here in the last five minutes, please remain seated until I've picked up all the exams.
9. Good luck!

Question	Points	Your Score
Q1	20	
Q2	20	
Q3	20	
Q4	20	
Q5	20	
TOTAL	100	

1. Suppose you are writing a contract between the producer of spliced ropes and the consumer, a parachute maker needing lines to attach a canopy to a harness.

   - The producer promises that the mean breaking strength of a shipment of the lines is $\mu = 100$ pounds, with $\sigma = 16$.
   - An independent lab will find $\bar{X}$ from a SRS of $n = 10$ lines to test $H_0 : \mu = (\mu_0 = 100)$ vs. $H_1 : \mu < \mu_0$.
   - A draft contract specifies $\bar{x}_{\text{critical}} = 97$. If $\bar{X}$ is below 97, H_0 is rejected and neither payment nor shipment occurs. If $\bar{X}$ is above or equal to 97, H_0 is not rejected and both payment and shipment occur. (Hint: A picture may help.)

   (a) Suppose you work for the producer (the splicing shop). If H_0 is true and the test nevertheless rejects H_0, the shipment of lines will be discarded, and you will not be paid. This happens with which probability? Mark your choice with an "X":

   ____ $P(\text{type I error}) = \alpha$,
   ____ $P(\text{type II error}) = \beta$, or
   ____ power $= 1 - \beta$

   ANSWER:

   $P(\text{type I error}) = \alpha$

   (b) In which direction would you like to move $\bar{x}_{\text{critical}} = 97$ to reduce your risk of not being paid in this situation? Mark your choice with an "X":

   ____ I want to move $\bar{x}_{\text{critical}}$ so that $\bar{x}_{\text{critical}} > 97$, or
   ____ I want to move $\bar{x}_{\text{critical}}$ so that $\bar{x}_{\text{critical}} < 97$

   ANSWER:

   The second choice is correct (so we'll reject H_0 when it is true less often).

   (c) Suppose you work for the consumer (the parachute maker). You can't use the lines if $\mu = 95$ (unless you redesign your parachute to use more of the weaker lines). If H_0 is false because $\mu = (\mu_A = 95)$, what is the probability that the test will not reject H_0? (In this case, you'll use a defective shipment of lines, and then sell defective parachutes.) Mark your choice with an "X":

   ____ $P(\text{type I error}) = \alpha_{(\mu_A=95)}$
   ____ $P(\text{type II error}) = \beta_{(\mu_A=95)}$
   ____ power $= 1 - \beta_{(\mu_A=95)}$

   ANSWER:

   $P(\text{type II error}) = \beta_{(\mu_A=95)}$

   (d) Which way would you like to move $\bar{x}_{\text{critical}}$ to decrease your risk? Mark your choice with an "X":

   ____ I want to move $\bar{x}_{\text{critical}}$ so that $\bar{x}_{\text{critical}} > 97$, or
   ____ I want to move $\bar{x}_{\text{critical}}$ so that $\bar{x}_{\text{critical}} < 97$

ANSWER:
The first choice is correct (so we'll reject H_0 when it is false more often).

(e) Suppose the draft contract is abandoned. What sample size is required to have level .01 and power .9 when the true population mean strength is 95 pounds? (Note: Increasing the sample size may resolve the tension between producer and consumer.)

ANSWER:
$n \approx \left(\frac{\sigma(z_{\alpha/2} + z_\beta)}{\mu_0 - \mu_A}\right)^2 = \left(\frac{16(z_{.005} + z_{.10})}{\mu_0 - \mu_A}\right)^2 = \left(\frac{16(2.575 + 1.285)}{100 - 95}\right)^2 = 152.6$, round up to $n = 153$.

(Note: this answer is mistaken, as the sample size formula I used is for $H_A : \mu \neq \mu_0$. This line strength test has $\mu < \mu_0$. I didn't derive the formula for this case in class, but the sample size required would use $z_\alpha = z_{.01} = 2.325$ instead of $z_{\alpha/2} = z_{.005} = 2.575$, giving $n = \left(\frac{16(2.325+1.285)}{100-95}\right)^2 = 133.4$, round up to $n = 134$.)

2. In a summer survey of Madison second grade children, 15 out of 48 had played outside in the past 24 hours.

   (a) Find a 90% confidence interval for the true proportion of students who played outside in the past 24 hours.

   ANSWER:

   The observed numbers of successes and failures, 15 and $33 = 48 - 15$, are both > 5. $P = \hat{\pi} = \frac{15}{48} = .3125$; margin $= z_{.05}\sqrt{\frac{.3125(1-.3125)}{48}} = 1.645(.0669) \approx .1101$; interval $=$ $.3125 \pm .1101$

   (b) Are the data strong evidence that the population proportion of children who played outside in the past 24 hours is less than 1/3? Do an appropriate test.

   ANSWER:

   - Hypotheses: $H_0 : \pi = 1/3$ vs. $H_A : \pi < 1/3$
   - Assumptions: We have a SRS. $n = 48$ and $\pi = 1/3 \implies$ the expected numbers of successes and failures are $16 = 48(1/3)$ and $32 = 48(1 - 1/3)$, which are both > 5. I'll use the test for one proportion.
   - Test statistic: $P = \hat{\pi} = \frac{15}{48} = .3125 \implies z = \frac{.3125-1/3}{\sqrt{\frac{(1/3)(1-1/3)}{48}}} \approx -0.31.$
   - P-value: $P(Z < -0.31) = .3783$
   - Conclusion: Do not reject H_0. The data are not strong evidence that the population proportion of children played outside in the past 24 hours is less than 1/3.

3. A simple random sample of 8 pills was taken and the time (in minutes) to dissolve in water was measured for each pill. Here are the data:

   24 72 294 18 54 66 120 48

   A few summary statistics include $n = 8$, $\bar{x} = 87$, and $s = 89.4$. Plots suggest that the population is not normal, so a bootstrap is used. 10000 resamples with replacement are taken, with a $\hat{t}$ calculated for each resample. Here are some quantiles of the resampling distribution of $\hat{t}$:

probability	.001	.005	.01	.05	.10	.50	.90	.95	.99	.995	.999
quantile	-10.2	-8.4	-7.6	-5.7	-3.7	-0.1	1.0	1.3	1.9	2.1	2.7

   (a) Find a 90% confidence interval for the population mean dissolve time for the pills.

   ANSWER:
   For a 90% interval, we have $1 - \alpha = .90 \implies \alpha = .10 \implies \alpha/2 = .05$ and we need the .05 and .95 quantiles, $\hat{t}_{.05} = -5.7$ and $\hat{t}_{.95} = 1.3$. Here is the interval:
   $\left(\bar{x} - \hat{t}_{(\alpha/2)}\frac{s}{\sqrt{n}}, \bar{x} - \hat{t}_{(1-\alpha/2)}\frac{s}{\sqrt{n}}\right) = \left(87 - (1.3)\frac{89.4}{\sqrt{8}}, 87 - (-5.7)\frac{89.4}{\sqrt{8}}\right) = (45.9, 267.2)$

   (b) Estimate the p-value for a bootstrap test of $H_0 : \mu = 100$ vs. $H_A : \mu < 100$, where μ is the population mean dissolve time, at level $\alpha = .05$. Draw a conclusion. (Hint: You cannot use R to estimate the p-value as we did in class. You can, nevertheless, estimate the p-value and draw a conclusion.)

   ANSWER:

$$\begin{aligned}
p\text{-value} &= P(T < t_{obs}) \\
&= P(T < \frac{87 - 100}{89.4/\sqrt{8}}) \\
&= P(T < -.41) \\
&= \text{between } .10 \text{ and } .50, \\
&\quad \text{because the .10 quantile is } -3.7 \text{ and the .50 quantile is } -0.1 \\
&> (\alpha = .05)
\end{aligned}$$

   Do not reject H_0. The data are not strong evidence that the population mean dissolve time is less than 100 seconds.

4. Each morning a hospital quality-control person checks two pulse oximeters by mounting them on a testing rig that simulates a stable patient. Experience shows that the oximeters give normally distributed readings with variances that match closely and change little, while the mean readings drift from one another over time. In one morning's check, simple random samples of 5 readings were taken from the first oximeter and of 7 readings from the second. The average oxygen level from the first oximeter was 98.3 with standard deviation 0.31, while the average from the second was 96.1 with standard deviation 0.29.

   - Find a 98% confidence interval for the difference in population mean readings from the two oximeters.

     ANSWER:
     We have normal populations and equal variances and so will use the 2-sample t confidence interval for a difference of two means.
     $s_p^2 = \frac{(5-1)0.31^2+(7-1)0.29^2}{5+7-2} = .0889$
     $1-\alpha = .98 \implies \alpha = .02 \implies \alpha/2 = .01$ and we need $t_{5+7-2,.01} = t_{10,.01} = 2.764$. The interval is $(\bar{X}-\bar{Y}) \pm t_{n_X+n_y-2}\sqrt{\frac{S_p^2}{n_X}+\frac{S_p^2}{n_Y}} = (98.3-96.1) \pm 2.764\sqrt{.0889/7+.0889/5} = 2.20 \pm 0.48$.

   - Test whether the population mean readings from the two oximeters are the same.

     ANSWER:
     We test $H_0 : \mu_X - \mu_Y = 0$ vs. $H_0 : \mu_X - \mu_Y \neq 0$.
     Our (2-sample t) test statistic is $t = \dfrac{(\bar{x}_1-\bar{x}_2)-\delta_0}{\sqrt{\frac{s_p^2}{n_1}+\frac{s_p^2}{n_2}}} = \dfrac{(98.3-96.1)-0}{\sqrt{.0889/5+.0889/7}} = 12.6$,
     so our p-value is $P(|T_{10}| > |12.6|) = 2P(T_{10} > 12.6) < 2(.001) = .002$, and we reject H_0. The data are strong evidence that the population mean readings are different.

5. A large hospital took a simple random sample of 10 babies delivered at the hospital whose mothers intended to breastfeed. Here are the number of days until weaning for each baby: 210, 217, 240, 270, 273, 289, 324, 330, 339, 530. Is the population median number of days to weaning less than 365 (one year)? Run an appropriate test.

ANSWER:

Test $H_0 : M = 365$ vs. $H_A : M < 365$ via a sign test.

Regarding assumptions, note that we have a SRS of babies delivered at the hospital whose mothers intended to breastfeed.

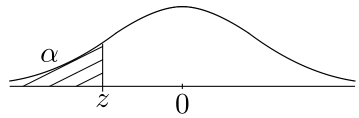
Let B = # babies in sample of 10 who are breastfed with days to weaning, minus 365 days, positive. This is the same as the number of days to weaning > 365. Under H_0, $B \sim Bin(n = 10, \pi = .5)$.

Our test statistic is $b = 1$.

The p-value is $P(B \leq 1) = P(B = 0) + P(B = 1) = \binom{10}{0}.5^0(1 - .5)^{10-0} + \binom{10}{1}.5^1(1 - .5)^{10-1} = .5^{10} + 10(.5^{10}) \approx 0.01$.

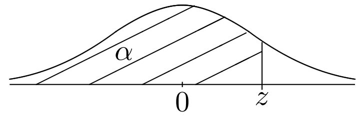
Since $0.01 < 0.05$, we reject H_0. The data are strong evidence the median #days to weaning is less than 365

For $z = a.bc$, look in row $a.b$ and column $.0c$ to find $P(Z < z)$. e.g.
For $z = 1.42$, look in row 1.4 and column .02 to find $P(Z < 1.42) = .9222$ (on next page).



Cumulative $N(0, 1)$ Distribution, $z \leq 0$

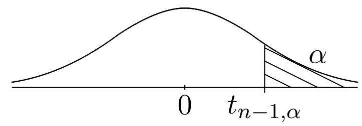
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641



Cumulative $N(0,1)$ Distribution, $z \geq 0$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	$\boxed{.9222}$	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999

Upper Tail Points $t_{n-1,\alpha}$ for the Student's t_{n-1} Distributions



degrees of freedom	right-tail area α						
$n-1$	.25	.10	.05	.025	.01	.005	.001
1	1.000	3.078	6.314	12.706	31.821	63.657	318.309
2	.816	1.886	2.920	4.303	6.965	9.925	22.327
3	.765	1.638	2.353	3.182	4.541	5.841	10.215
4	.741	1.533	2.132	2.776	3.747	4.604	7.173
5	.727	1.476	2.015	2.571	3.365	4.032	5.893
6	.718	1.440	1.943	2.447	3.143	3.707	5.208
7	.711	1.415	1.895	2.365	2.998	3.499	4.785
8	.706	1.397	1.860	2.306	2.896	3.355	4.501
9	.703	1.383	1.833	2.262	2.821	3.250	4.297
10	.700	1.372	1.812	2.228	2.764	3.169	4.144
11	.697	1.363	1.796	2.201	2.718	3.106	4.025
12	.695	1.356	1.782	2.179	2.681	3.055	3.930
13	.694	1.350	1.771	2.160	2.650	3.012	3.852
14	.692	1.345	1.761	2.145	2.624	2.977	3.787
15	.691	1.341	1.753	2.131	2.602	2.947	3.733
16	.690	1.337	1.746	2.120	2.583	2.921	3.686
17	.689	1.333	1.740	2.110	2.567	2.898	3.646
18	.688	1.330	1.734	2.101	2.552	2.878	3.610
19	.688	1.328	1.729	2.093	2.539	2.861	3.579
20	.687	1.325	1.725	2.086	2.528	2.845	3.552
21	.686	1.323	1.721	2.080	2.518	2.831	3.527
22	.686	1.321	1.717	2.074	2.508	2.819	3.505
23	.685	1.319	1.714	2.069	2.500	2.807	3.485
24	.685	1.318	1.711	2.064	2.492	2.797	3.467
25	.684	1.316	1.708	2.060	2.485	2.787	3.450
26	.684	1.315	1.706	2.056	2.479	2.779	3.435
27	.684	1.314	1.703	2.052	2.473	2.771	3.421
28	.683	1.313	1.701	2.048	2.467	2.763	3.408
29	.683	1.311	1.699	2.045	2.462	2.756	3.396
30	.683	1.310	1.697	2.042	2.457	2.750	3.385
40	.681	1.303	1.684	2.021	2.423	2.704	3.307
60	.679	1.296	1.671	2.000	2.390	2.660	3.232
100	.677	1.290	1.660	1.984	2.364	2.626	3.174
∞	.674	1.282	1.645	1.960	2.326	2.576	3.090