STAT 371 Exam 2 NetID (mine is "jgillett" from "jgillett@wisc.edu"): ____________

Last name: ______________________________ First name: ________________________

Discussion (check one):

Lecture 002 TuTh 1:00-2:15 Soils 270

____ 321 Tu 3:30-4:20 Grainger 1185 with Trane, Ralph and Park, Chan

____ 322 Tu 4:35-5:25 Social Sciences 5231 with Trane, Ralph and Park, Chan

____ 323 We 9:55-10:45 Sterling 1313 with Trane, Ralph and Liu, Hongzhi

Lecture 003 TuTh 2:30-3:45 Van Vleck B130

____ 331 Tu 4:35-5:25 Psychology 121 with White, David and Pritchard, Nathaniel

____ 332 We 9:55-10:45 Psychology 103 with White, David and Pritchard, Nathaniel

____ 333 We 11:00-11:50 Ingraham 22 with White, David and Pritchard, Nathaniel

Instructions:

1. Do not open the exam until I say "go."
2. Put away everything except a pencil, a calculator, and your two one-page (two sides each) notes sheets.
3. Show your work. Correct answers without enough work may receive no credit.
4. Find the required table(s) at the end of the packet. You may tear the tables sheet(s) free.
5. If a question is ambiguous, resolve the ambiguity in writing. We will consider grading accordingly.
6. The exam ends when I call time. If you continue writing after I call time, you risk a penalty. (The alternative, that you get more time than your peers, is unfair.)
7. You are welcome to turn your exam in to me before I call time. However, if you are still here in the last five minutes, please remain seated until I've called time.

Question	Points	Deduction
Q0 (cover)	1	
Q0 (smile)	2	0
Q1	12	
Q2	21	
Q3	12	
Q4	20	
Q5	14	
Q6	18	
Total	100	

0. (This optional question is worth 2 free points, even for no answer. Do not consider it until you're done with the exam.) Write something, if you wish, to make your graders smile.

1. A public health officer surveys takes a simple random sample of a rural county's population and visits each person in the sample to discuss a vaccine program. He finds that 29 of the 40 people sampled had heard about the program before his visit. Find a 90% confidence interval for the county population proportion of people who have heard about the program.

   (a) Which interval is appropriate? Mark the best answer with "X".

      i. _______ Z interval for one mean
      ii. _______ t interval for one mean
      iii. _______ bootstrap interval for one mean
      iv. _______ Z interval for one proportion
      v. _______ t interval for one proportion
      vi. _______ t interval for a difference of two means

   ANSWER:
   Z interval for one proportion

   (b) Are the assumptions for your interval met? Why or why not?

   ANSWER:
   We need and have a simple random sample. We need the numbers of successes and failures each greater than 5, and 29 and $11 = 40 - 29$ are both greater than 5.

   (c) Find the value of confidence interval in the form "center $\pm$ error margin".

   ANSWER:
   We have $x = 29$ successes in $n = 40$ trials, for a sample proportion $p = \frac{29}{40} = .725$. For 90% confidence, we have $1 - \alpha = .90 \implies \alpha = .10$ and we need $z_{.05} = 1.645$. The interval is $P \pm z_{\alpha/2}\sqrt{\frac{P(1-P)}{n}} = .725 \pm 1.645\sqrt{\frac{.725(1-.725)}{40}} = .725 \pm .116$.

2. Circle the best answer after each question about confidence intervals and tests.

(a) What effect does increasing a test's probability of making a type II error have on the test's power?

i. It decreases it.

ii. It increases it.

ANSWER:

(i) It decreases it, since power $= 1 - \beta$, and β is the probability of a type II error.

(b) Consider making a bootstrap confidence interval for an unknown mean μ of the form $\left(\bar{x} - \hat{t}_{(\alpha/2)}\frac{s}{\sqrt{n}}, \bar{x} - \hat{t}_{(1-\alpha/2)}\frac{s}{\sqrt{n}}\right)$. The bootstrap procedure, as contrasted to the Z or Student's t procedure, helps with which part of the interval?

i. Using the sample mean $\bar{x}$ to estimate the unknown population mean μ.

ii. Using the sample standard deviation s to estimate the unknown population standard deviation σ.

iii. Using the resampled $\hat{t}$ values to estimate the unknown sampling distribution of $T = \dfrac{\bar{X} - \mu}{S/\sqrt{n}}$.

iv. Using the resampled $\hat{t}$ values to estimate the unknown confidence level $1 - \alpha$.

ANSWER:

Regarding (i) and (ii), the bootstrap and Z and Student's t procedures all do the same thing. (iii) is correct: we use the $\hat{t}$ values to calculate $\hat{t}_{(1-\alpha/2)}$ and $\hat{t}_{(\alpha/2)}$ as estimates of the corresponding quantiles of the unknown distribution of T. (iv) is wrong because $1 - \alpha$ is a given confidence level, so there's no need to estimate it.

(c) Consider running a bootstrap hypothesis test for $H_0 : \mu = \mu_0$ against some H_A. The bootstrap procedure, as contrasted to the Z or Student's t procedure, helps with which part of the test?

i. Using the sample mean $\bar{x}$ and the sample standard deviation s to find the test statistic, $t_{\text{obs}} = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

ii. Deciding which alternate hypothesis from among $H_A : \mu > 0$, $H_A : \mu < 0$, and $H_A : \mu \neq 0$ is most appropriate.

iii. Using the resampled $\hat{t}$ values to estimate the p-value of the test.

iv. Using the resampled $\hat{t}$ values to estimate the unknown significance level α.

ANSWER:

Regarding (i) and (ii), the bootstrap and Z and Student's t procedures all do the same thing. (iii) is correct: we use the $\hat{t}$ values to calculate the p-value as a proportion of the $\hat{t}$ values, estimating the corresponding area of the unknown distribution of $T = \dfrac{\bar{X} - \mu}{S/\sqrt{n}}$. (iv) is wrong because α is a given in specifying the test, so there's no need to estimate it.

(d) Consider a Z test at level α of $H_0 : \mu = \mu_0$ vs. $H_A : \mu \neq \mu_0$ where σ is known. Suppose we do not reject H_0. Mark each true statement with "X".

i. _______ We do not reject H_0 at level $\alpha/2$ either.

ii. _______ The rejection region corresponding to level α contains the rejection region corresponding to level $\alpha/2$.

iii. _______ The confidence interval $\bar{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ contains μ_0.

iv. _______ The power of the test is $1 - \alpha$.

ANSWER:

(i), (ii), and (iii) are true. (iv) is false.

3. A maple syrup producer is visiting properties covered by maple forests to choose one that produces sap with high sugar content. On one property he plans to use a sample of 40 trees to test $H_0 : \mu = 3$ vs. $H_A : \mu > 3$, where μ is the population average sugar content (in grams of sugar per 100 grams of sap) across the trees on that property. He decides he will treat his sample standard deviation as a good estimate of σ, the population standard deviation, and run a Z test using significance level $\alpha = .04$.

He samples 40 trees and finds their average sugar content is 3.2 with standard deviation 0.5.

(a) Suppose the true mean sugar content is $\mu = 3.1$. Find the power of his test.

ANSWER:

$\text{power}_{\mu_A=3.1} = P\left(Z < \frac{|\mu_0 - \mu_A|}{\sigma/\sqrt{n}} - z_\alpha\right) = P\left(Z < \frac{|3 - 3.1|}{.5/\sqrt{40}} - 1.755\right) = P(Z < -0.49) =$ $.3121$

(b) Find the value of the test statistic.

ANSWER:

$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{3.2-3}{.5/\sqrt{40}} \approx 2.53$

(c) For the test statistic $z = 2.17$ (not the value you just found), find the p-value of the test.

ANSWER:

p-value $= P(Z > 2.17) = .015$

(d) Suppose the p-value is 0.01 (not the value you just found). What conclusion should he draw at the given significance level?

ANSWER:

Since (p-value $= .01$) $<$ ($\alpha = .04$), reject H_0.

4. A nitrile rubber glove maker took a simple random sample of 10 of its supply of gloves and tested them for abrasion resistance. Here are the number of minutes on an abrading machine until failure: 1, 1, 1, 2, 2, 2, 4, 4, 4, 11. Is the population median number of minutes on the machine until failure significantly less than 15 at level $\alpha = .05$? Run an appropriate test.

   (a) What hypotheses and which test are suitable?

   ANSWER:

   Test $H_0 : M = 15$ vs. $H_A : M < 15$ via a sign test.

   (b) What assumptions are required?

   ANSWER:

   We need a simple random sample; the problem says we have one from the supply of gloves.

   (c) What is the value of the test statistic?

   ANSWER:

   Let $B = \#$ gloves in the sample of 10 whose time to failure is more than 15 minutes. Under H_0, $B \sim Bin(n = 10, \pi = .5)$.

   Our test statistic is $b = 0$.

   (d) Find the p-value corresponding to a test statistic value of 0 for the alternative hypothesis constructed using "<". (This may not be the alternative hypothesis or test statistic value you found in the previous steps.)

   ANSWER:

   The p-value is $P(B \leq 0) = \binom{10}{0} .5^0 (1 - .5)^{10-0} = .5^{10} \approx .001$.

   (e) Suppose the p-value is .12 (this may not be the value you just found). Do you reject H_0? yes / no (circle one)

   ANSWER:

   No. Since .12 is not less than .05, do not reject H_0. The data are not strong evidence the median #minutes to failure is less than 15.

5. This problem spans two pages. On this page is the description of the bootstrapping algorithm from lecture; its steps are labeled (a) through (g). On the next page is R code used to implement the algorithm; it is divided into seven code chunks labeled (A) through (G).

   For each of the seven algorithm steps, write its label, one of (a) through (g), into one of the seven blanks (A) through (G) to indicate that step is implemented by the corresponding R code fragment.

Write your answers here:	Here is a sample (wrong) answer:
(A) ____________ ANSWER: a	(A) g
(B) ____________ ANSWER: e	(B) f
(C) ____________ ANSWER: b	(C) e
(D) ____________ ANSWER: c	(D) d
(E) ____________ ANSWER: d	(E) c
(F) ____________ ANSWER: f	(F) b
(G) ____________ ANSWER: g	(G) a

   (a) Collect one simple random sample of size n from the population. Compute the sample mean, $\bar{x}$ (an estimate of the population mean, μ) and the sample standard deviation, s (an estimate of the population standard deviation, σ).

   (b) Draw a random sample of size n, with replacement, from the data. Call these observations x_1^*, x_2^*, ..., x_n^*. Some data may appear more than once in this resampling, and some not at all.

   (c) Compute the mean and standard deviation of the resampled data. Call these $\bar{x}^*$ and s^*.

   (d) Compute the statistic $\hat{t} = \dfrac{\bar{x}^* - \bar{x}}{s^*/\sqrt{n}}$

   (e) Repeat steps 2-4 a large number of times, accumulating many $\hat{t}$'s. They approximate the (unknown) sampling distribution of $T = \frac{\bar{X}-\mu}{S/\sqrt{n}}$.

   (f) To find a $(100\%)(1-\alpha)$ confidence interval for μ, find the $1-\alpha/2$ and $\alpha/2$ upper *critical values* of the approximate sampling distribution, calling them $\hat{t}_{(1-\alpha/2)}$ and $\hat{t}_{(\alpha/2)}$. The bootstrap $100(1-\alpha)\%$ confidence interval is $\left(\bar{x} - \hat{t}_{(\alpha/2)}\dfrac{s}{\sqrt{n}}, \bar{x} - \hat{t}_{(1-\alpha/2)}\dfrac{s}{\sqrt{n}}\right)$.

   (g) To test $H_0 : \mu = \mu_0$, compute $t_{\text{obs}} = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$. Find the p-value, an area under the approximate sampling distribution density curve given by $\frac{m}{B}$, where m depends on H_A:
   $H_A : \mu > \mu_0 \implies m$ is the number of values of $\hat{t}$ for which $\hat{t} > t_{\text{obs}}$
   $H_A : \mu < \mu_0 \implies m$ is the number of values of $\hat{t}$ for which $\hat{t} < t_{\text{obs}}$
   $H_A : \mu \neq \mu_0 \implies m$ is the number of values of $\hat{t}$ for which $\hat{t} < -|t_{\text{obs}}|$ or $\hat{t} > |t_{\text{obs}}|$
   Draw a conclusion as usual: $\begin{cases} p\text{-value} \leq \alpha \text{ (where } \alpha \text{ is the level, .05 by default)} \implies \text{reject } H_0 \\ p\text{-value} > \alpha \implies \text{retain } H_0 \text{ as plausible} \end{cases}$

(A)

```
data = c(29, 30, 53, 75, 34, 21, 12, 58, 117, 119, 115, 134, 253, 289, 287)
n = length(data)
x.bar = mean(data)
s = sd(data)
```

```
bootstrap = function(x, n.boot) {
  n = length(x)
  x.bar = mean(x)
  t.hat = numeric(n.boot)
```

(B)

```
  for(i in 1:n.boot) {
```

(C)

```
    x.star = sample(x, size=n, replace=TRUE)
```

(D)

```
    x.bar.star = mean(x.star)
    s.star = sd(x.star)
```

(E)

```
    t.hat[i] = (x.bar.star - x.bar) / (s.star / sqrt(n))
  }
  return(t.hat)
}

B = 5000
t.hats = bootstrap(data, B)
```

(F)

```
t.lower = quantile(t.hats, probs=.025) # This is our t_{1 - alpha.2}.
t.upper = quantile(t.hats, probs=.975) # This is our t_{alpha/2}.
cat(sep="", "t.lower=", t.lower, ", t.upper=", t.upper, "\n")
ci.low  = x.bar - t.upper * s / sqrt(n)
ci.high = x.bar - t.lower * s / sqrt(n)
cat(sep="", "confidence interval: (", ci.low, ", ", ci.high, ")\n")
```

(G)

```
mu.0 = 75
t.obs = (x.bar - mu.0) / (s / sqrt(n))
cat(sep="", "t.obs=", t.obs, "\n")
m.left = sum(t.hats < t.obs) # This is for H_A: mu < mu_0.
p.value.left = m.left / B
cat(sep="", "m.left=", m.left, ", B=", B, ", p.value.left=", p.value.left, "\n")
m.right = sum(t.hats > t.obs) # This is for H_A: mu > mu_0.
p.value.right = m.right / B
cat(sep="", "m.right=", m.right, ", B=", B, ", p.value.right=", p.value.right, "\n")

# This is for H_A: mu != mu_0. ("!=" means "is not equal to.")
m.left.abs = sum(t.hats < -abs(t.obs))
m.right.abs = sum(t.hats > abs(t.obs))
p.value.two.sided = (m.left.abs + m.right.abs) / B
cat(sep="", "m.left.abs=", m.left.abs, ", m.right.abs=", m.right.abs,
    ", B=", B, ", p.value.two.sided=", p.value.two.sided, "\n")
```

6. A couple is considering developing an energy plantation on their land by planting trees for firewood. They visit an existing firewood supplier and purchases air dried hickory logs and oak logs that they decide to treat as independent simple random samples from the local forests. They cuts and pack the logs into many 1-cubic foot boxes for weighing. Here are the resulting weights in pounds:

tree	sample mean	sample standard deviation	sample size
hickory	51	3	40
oak	44	2.5	154

QQ plots are compatible with normal populations. Consider a test to decide whether these data strong evidence, at significance level 0.05, that cubic feet of wood from hickory logs are heavier than those from oak logs.

(a) What hypotheses are suitable?

ANSWER:

$H_0 : \mu_{\text{hickory}} - \mu_{\text{oak}} = 0$ vs. $H_A : \mu_{\text{hickory}} - \mu_{\text{oak}} > 0$

(b) What assumptions are required?

ANSWER:

We have (well, they suppose) independent simple random samples from normal populations. The sample variances, $3^2 = 9$ and $2.5^2 = 6.25$, are within a factor of 2, so we assume equal population variances and use the 2-sample t-test.

(c) What is the value of the test statistic?

ANSWER:

The pooled sample variance is $s_p^2 = \frac{(40-1)3^2+(154-1)2.5^2}{40+154-2} \approx 6.809$.

$t = \frac{(51-44)-0}{\sqrt{\frac{6.809}{40}+\frac{6.809}{154}}} = 15.12$

(d) Find the p-value corresponding to a test statistic value of 2.326 for the alternative hypothesis constructed using ">". (This may not be the alternative hypothesis or test statistic value you found in the previous steps.)

ANSWER:

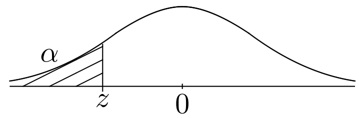
P-value $= P(T_{40+154-2} > 2.326) = P(T_{192} > 2.326) \approx P(T_\infty > 2.326) \approx .01$.

(e) Do you reject H_0? yes / no (circle one)

ANSWER:

yes

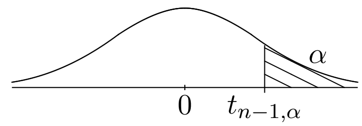
For $z = a.bc$, look in row $a.b$ and column $.0c$ to find $P(Z < z)$. e.g.
For $z = 1.42$, look in row 1.4 and column .02 to find $P(Z < 1.42) = .9222$ (on next page).



Cumulative $N(0,1)$ Distribution, $z \le 0$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Upper Tail Points $t_{n-1,\alpha}$ for the Student's t_{n-1} Distributions



degrees of freedom	right-tail area α						
$n-1$	.25	.10	.05	.025	.01	.005	.001
1	1.000	3.078	6.314	12.706	31.821	63.657	318.309
2	.816	1.886	2.920	4.303	6.965	9.925	22.327
3	.765	1.638	2.353	3.182	4.541	5.841	10.215
4	.741	1.533	2.132	2.776	3.747	4.604	7.173
5	.727	1.476	2.015	2.571	3.365	4.032	5.893
6	.718	1.440	1.943	2.447	3.143	3.707	5.208
7	.711	1.415	1.895	2.365	2.998	3.499	4.785
8	.706	1.397	1.860	2.306	2.896	3.355	4.501
9	.703	1.383	1.833	2.262	2.821	3.250	4.297
10	.700	1.372	1.812	2.228	2.764	3.169	4.144
11	.697	1.363	1.796	2.201	2.718	3.106	4.025
12	.695	1.356	1.782	2.179	2.681	3.055	3.930
13	.694	1.350	1.771	2.160	2.650	3.012	3.852
14	.692	1.345	1.761	2.145	2.624	2.977	3.787
15	.691	1.341	1.753	2.131	2.602	2.947	3.733
16	.690	1.337	1.746	2.120	2.583	2.921	3.686
17	.689	1.333	1.740	2.110	2.567	2.898	3.646
18	.688	1.330	1.734	2.101	2.552	2.878	3.610
19	.688	1.328	1.729	2.093	2.539	2.861	3.579
20	.687	1.325	1.725	2.086	2.528	2.845	3.552
21	.686	1.323	1.721	2.080	2.518	2.831	3.527
22	.686	1.321	1.717	2.074	2.508	2.819	3.505
23	.685	1.319	1.714	2.069	2.500	2.807	3.485
24	.685	1.318	1.711	2.064	2.492	2.797	3.467
25	.684	1.316	1.708	2.060	2.485	2.787	3.450
26	.684	1.315	1.706	2.056	2.479	2.779	3.435
27	.684	1.314	1.703	2.052	2.473	2.771	3.421
28	.683	1.313	1.701	2.048	2.467	2.763	3.408
29	.683	1.311	1.699	2.045	2.462	2.756	3.396
30	.683	1.310	1.697	2.042	2.457	2.750	3.385
40	.681	1.303	1.684	2.021	2.423	2.704	3.307
60	.679	1.296	1.671	2.000	2.390	2.660	3.232
100	.677	1.290	1.660	1.984	2.364	2.626	3.174
∞	.674	1.282	1.645	1.960	2.326	2.576	3.090