Lecture 8: Functional Programming with `itertools` and `functools`
Functional Programming

In the last lecture, we saw ideas from object oriented programming

“Everything is an object”
Every operation is the responsibility of some class/object
Use side effects to our advantage (e.g., modifying attributes)

In **functional programming**, functions are the central concept, not objects

“Everything is a function”, “data is immutable”
Avoid side effects at all costs
Use pure functions (and “meta-functions”) as much as possible
Iterators (or their equivalents) become hugely important
Iterators

An iterator is an object that represents a “data stream”

Supports method `__next__()`:
returns next element of the stream/sequence
raises `StopIteration` error when there are no more elements left
Iterators

An iterator is an object that represents a “data stream”

Supports method `__next__()`:
- returns next element of the stream/sequence
- raises `StopIteration` error when there are no more elements left

```
class Squares():
    '''Iterator over the squares.'''
    def __init__(self):
        self.n = 0
    def __next__(self):
        (self.n, k) = (self.n+1, self.n)
        return (k*k)

s = Squares()
[next(s) for _ in range(10)]
```

`__next__()` is the important point, here. It returns a value, the next square.

`next(iter)` is equivalent to calling `__next__()`. Variable `_` in the list comprehension is a placeholder, tells Python to ignore the value.
Iterators

Lists are not iterators, but we can turn a list into an iterator by calling `iter()` on it. Thus, lists are iterable, meaning that it is possible to obtain an iterator over their elements.

https://docs.python.org/3/glossary.html#term-iterable

From the documentation: “When an iterable object is passed as an argument to the built-in function `iter()`, it returns an iterator for the object. This iterator is good for one pass over the set of values. When using iterables, it is usually not necessary to call `iter()` or deal with iterator objects yourself. The `for` statement does that automatically for you, creating a temporary unnamed variable to hold the iterator for the duration of the loop.”
Lists are **not** iterators, so we first have to turn the list \( t \) into an iterator using the function `iter()`.

Now, each time we call `next()`, we get the next element in the list. **Reminder:** `next(iter)` and `iter.__next__()` are equivalent.

Once we run out of elements, we get an error.
You are already familiar with iterators from previous lectures. When you ask Python to traverse an object `obj` with a for-loop, Python calls `iter(obj)` to obtain an iterator over the elements of `obj`.

These two for-loops are equivalent. The first one hides the call to `iter()` from you, whereas in the second, we are doing the work that Python would otherwise do for us by casting `t` to an iterator.
You are already familiar with iterators from previous lectures. When you ask Python to traverse an object `obj` with a for-loop, Python calls `iter(obj)` to obtain an iterator over the elements of `obj`.

These two for-loops are equivalent. The first one hides the call to `iter()` from you, whereas in the second, we are doing the work that Python would otherwise do for us by casting `t` to an iterator.

A useful note from the documentation: “There is a subtlety when the sequence is being modified by the loop (this can only occur for mutable sequences, i.e. lists). An internal counter is used to keep track of which item is used next, and this is incremented on each iteration. When this counter has reached the length of the sequence the loop terminates. This means that if the suite deletes the current (or a previous) item from the sequence, the next item will be skipped (since it gets the index of the current item which has already been treated). Likewise, if the suite inserts an item in the sequence before the current item, the current item will be treated again the next time through the loop.”
Iterators

If we try to iterate over an object that is not iterable, we’re going to get an error.

```
class dummy():
    '''Class that is not iterable,
    because it has neither __next__() nor __iter__().'''

d = dummy()
for x in d:
    print(x)
```

Objects of class `dummy` have neither `__iter__()` (i.e., doesn’t support `iter()`) nor `__next__()` , so iteration is hopeless. When we try to iterate, Python is going to raise a `TypeError`.

```
Traceback (most recent call last)
<ipython-input-30-fc084e213893> in <module>()
    5
d = dummy()
----> 7 for x in d:
    8    print(x)

TypeError: 'dummy' object is not iterable`
Iterators

```python
class Squares:
    '''Iterator over the squares.''
    def __init__(self):
        self.n = 0
    def __next__(self):
        (self.n, k) = (self.n+1, self.n)
        return(k*k)
s = Squares()
for x in s:
    print(x)
```

Merely being an iterator isn’t enough, either! `for X in Y` requires that object `Y` be iterable.
Iterators

Iterable means that an object has the `__iter__()` method, which returns an iterator. So `__iter__()` returns a new object that supports `__next__()`.

```
class Squares:
    '''Iterator over the squares.'''
    def __init__(self):
        self.n = 0
    def __next__(self):
        (self.n, k) = (self.n+1, self.n)
        return(k*k)
    def __iter__(self):
        return(self)

s = Squares()
for x in s:
    print(x)
```

Now `Squares` supports `__iter__()` (it just returns itself!), so Python allows us to iterate over it.

This is an infinite loop. Don’t try this at home.
Iterators

We can turn an iterator back into a list, tuple, etc. **Caution:** if you have an iterator like our `Squares` example earlier, this list is infinite and you'll just run out of memory.

```
1  tl = ['cat', 'dog', 'bird', 'goat']
2  tl_iter = iter(tl)
3  t2 = list(tl_iter)
4  tl == t2
True
```

```
1  tl is t2
False
```

Many built-in functions work on iterators. e.g., `max`, `min`, `sum`, work on any iterator (provided elements support the operation); `in` operator will also work on any iterator.

**Warning:** Once again, care must be taken if the iterator is infinite.
List Comprehensions and Generator Expressions

Recall that a list comprehension creates a list from an iterable.

```
def square(k):
    return(k*k)
[square(x) for x in range(17) if x%2==0]
```

List comprehension computes and returns the whole list. What if the iterable were infinite? Then this list comprehension would never return!

```
s = Squares()
[x**2 for x in s]
```

This list comprehension is going to be infinite! But I really ought to be able to get an iterator over the squares of the elements of Squares object s...

```
sqgen = (x**2 for x in s)
sqgen
```

This is the motivation for generator expressions. Generator expressions are like list comprehensions, but they create an iterator rather than a list.

Generator expressions are written like list comprehensions, but with parentheses instead of square brackets.
Generators

Related to generator expressions are **generators**. Provide a simple way to write iterators (avoids having to create a new class)

```python
1 def harmonic(n):
2     return sum([1/k for k in range(1,n+1)])
3 harmonic(10)
2.9289682539682538
```

Each time we call this function, a local namespace is created, we do a bunch of work there, and then all that work disappears when the namespace is destroyed.

```python
1 def harmonic():
2     (h,n) = (0,1)
3     while True:
4         (h,n) = (h+1/n, n+1)
5         yield h
6     h = harmonic()
7 [next(h) for _ in range(3)]
[1.0, 1.5, 1.8333333333333333]
```

Alternatively, we can write `harmonic` as a **generator**. Generators work like functions, but they maintain internal state, and they **yield** instead of **return**. Each time a generator gets called, it runs until it encounters a **yield** statement or reaches the end of the **def** block.

Generators

```python
def harmonic():
    (h, n) = (0, 1)
    while True:
        (h, n) = (h + 1/n, n + 1)
        yield h
    h = harmonic()
    h
```

<generator object harmonic at 0x1053b9fc0>

```python
next(h)
1.0
```

```python
next(h)
1.5
```

```python
next(h)
1.8333333333333333
```
Generators

Python sees the `yield` keyword and determines that this should be a generator definition rather than a function definition.

Create a new `harmonic` generator. Inside this object, Python keeps track of where in the `def` code we are. So far, no code has been run.
Generators

Python sees the `yield` keyword and determines that this should be a generator definition rather than a function definition.

```python
def harmonic():
    (h, n) = (0, 1)
    while True:
        (h, n) = (h+1/n, n+1)
        yield h

h = harmonic()  
```

<generator object harmonic at 0x1053b9fc0>

Each time we call `next`, Python runs the code in `h` from where it left off until it encounters a `yield` statement.

```
1 next(h)  
1.0

1 next(h)  
1.5

1 next(h)  
1.8333333333333333
```
Generators

Python sees the `yield` keyword and determines that this should be a generator definition rather than a function definition.

```
def harmonic():
    (h, n) = (0, 1)
    while True:
        (h, n) = (h+1/n, n+1)
        yield h
    h = harmonic()
    h
<generator object harmonic at 0x1053b9fc0>
```

Each time we call `next`, Python runs the code in `h` from where it left off until it encounters a `yield` statement.

```
next(h)
1.0
next(h)
1.5
next(h)
1.8333333333333333
```
Generators

```python
def harmonic():
    (h,n) = (0,1)
    while True:
        (h,n) = (h+1/n, n+1)
        yield h
    h = harmonic()
    h

<generator object harmonic at 0x1053b9fc0>
```

Python sees the `yield` keyword and determines that this should be a generator definition rather than a function definition.

Each time we call `next`, Python runs the code in `h` from where it left off until it encounters a `yield` statement.
Generators

Python sees the `yield` keyword and determines that this should be a generator definition rather than a function definition.

```
def harmonic():
    (h,n) = (0,1)
    while True:
        (h,n) = (h+1/n, n+1)
        yield h
    h = harmonic()
```

<generator object harmonic at 0x1053b9fc0>

Each time we call `next`, Python runs the code in `h` from where it left off until it encounters a `yield` statement.

```
1  next(h)
1.0
```

```
1  next(h)
1.5
```

```
1  next(h)
1.8333333333333333
```
Generators

Python sees the `yield` keyword and determines that this should be a generator definition rather than a function definition.

```python
def harmonic():
    (h,n) = (0,1)
    while True:
        (h,n) = (h+1/n, n+1)
        yield h
    h = harmonic()
```

If/when we run out of `yield` statements (i.e., because we reach the end of the definition block), the generator returns a `StopIteration` error, as required of an iterator (not shown here).

```python
next(h)
```

```
1.0
```

```python
next(h)
```

```
1.5
```

```python
next(h)
```

```
1.8333333333333333
```
Generators

Generators supply a few more bells and whistles

- Ability to pass values *into* the generator to modify behavior
- Can make generators both produce and consume information
  - **Coroutines** as opposed to **subroutines**

See generator documentation for more:

https://docs.python.org/3/reference/expressions.html#generator-iterator-methods
zip, revisited

Recall that `zip` takes two or more iterables and returns an iterator over tuples.

Here are two infinite iterators, and we `zip` them. So `z` should also be an infinite iterator. But this expression doesn’t result in an infinite evaluation...

The trick is that `zip` uses **lazy evaluation**. Rather than trying to build all the tuples right when we call `zip`, Python is lazy. It only builds tuples as we ask for them! We’ll see this plenty more in this course. [https://en.wikipedia.org/wiki/Lazy_evaluation](https://en.wikipedia.org/wiki/Lazy_evaluation)
Map and Filter

Recall:

**map** operation applies a function to every element of a sequence
Yields a new, transformed sequence

**filter** operation removes from a sequence all elements failing some condition
Again, yields a new, filtered sequence
Map

We saw how to achieve a map operation using list comprehensions

But there’s also the Python `map` function:

```python
def square_plus1(x):
    return x**2+1
map(square_plus1, range(10))

list(map(square_plus1, range(10)))
```

From the documentation:
`map(function, iterable, ...)`
Return an iterator that applies `function` to every item of `iterable`, yielding the results.

`map` and `range` both produce special kinds of iterators.
The first argument to `map` is a function; remaining arguments are one or more iterables.

Number of iterables and number of function arguments must agree!
Aside: \texttt{lambda} expressions

Lambda expressions let you define functions without using a \texttt{def} statement. Called an \textbf{in-line function} or \textbf{anonymous function}. Name is a reference to \textit{lambda calculus}, a concept from symbolic logic.

Define a function, then pass it to \texttt{map}.

Alternatively, define an equivalent function \textbf{in-line}, using a \texttt{lambda} statement.

A lambda expression returns a function, so \texttt{my_square} and \texttt{lambda x: x**2} are, in a certain sense, equivalent.
Aside: lambda expressions

Arguments of the function are listed before the colon. So this function takes a single argument...

...while this one takes four.
Aside: **lambda expressions**

Return value of the function is listed on the right of the colon. So this function returns the square of its input plus 1.

...and this one returns a Boolean stating whether or not the four numbers satisfy Fermat’s last theorem.

---

```python
lambda x : x**2 + 1
<function __main__.<lambda>>
```

```python
lambda x,y,z,n : x**n + y**n == z**n
<function __main__.<lambda>>
```

```python
(lambda x,y,z,n : x**n + y**n == z**n)(3,4,5,2)
True
```

```python
(lambda x,y,z,n : x**n + y**n == z**n)(13,17,19,42)
False
```

```python
my_square
<function __main__.my_square>
```

[https://en.wikipedia.org/wiki/Fermat%27s_Last_Theorem](https://en.wikipedia.org/wiki/Fermat%27s_Last_Theorem)
Lambda expressions return actual functions, which we can apply to inputs.

Function names are stored in an attribute __name__. Since lambda expressions yield anonymous functions, they all have the generic name `<lambda>`. 
Aside: `lambda` expressions

Lambda expressions can be used anywhere you would use a function. Note that the term **anonymous function** makes sense: the lambda expression defines a function, but it never gets a variable name (unless we assign it to something, like in the ‘goat’ example to the left).

```
1 f = lambda x : x+'goat'
2 f('cat')
'catgoat'
1 (lambda x : 2*x)(21)
42
1 list(map(lambda x: x**2, range(1,10)))
[1, 4, 9, 16, 25, 36, 49, 64, 81]
```
First-class functions

The fact that we can have variables whose values are functions is actually quite special. We say that Python has **first-class functions**. That is, functions are perfectly reasonable values for a variable to have.

You've seen these ideas before if you've used R’s `tapply` (or similar), MATLAB’s function handles, C/C++ function pointers, etc.
The list filter expression also has an analogous function, `filter`.

`filter` takes a Boolean function and an iterator and returns an iterator of only the elements that evaluated to `True`.

Returns its own special iterator.

Second argument to `filter` (and `map`) can be any iterator. Here we are filtering a generator.
Filter

It's often more convenient to just use a lambda expression in-line instead of defining a Boolean function elsewhere.

Lambda expressions don't support scatter/gather, so you have to use this kind of pattern to process tuples. Worry not! Another Python module does support this, and we'll see it in a few slides.
What about reduce?

We saw map and filter earlier, but we can’t have MapReduce without reduce.

Reduce operations reduce an iterator (i.e., a sequence) to a single element. Sum is a good example of a reduce function.

```
1  sum(range(1,11))
```

55

**functools** contains a bunch of useful functional programming functions, including reduce.

```
1  import functools
2  functools.reduce(lambda x,y: x+y, range(1,11))
```

55

A reduce operation takes a function and a sequence, and returns a single object (typically of the same type as the elements of the sequence). `sum()` is a good example of a reduce operation, but it’s hardly the only one.

**functools.reduce** takes a function and an iterator and performs a reduce operation on the iterator using the function.
Reduce operations

Three fundamental pieces:

- **function**
  \[ f(x, y) = x + y \]

- **iterable**
  [2, 3, 5, 8, 1, 1, 7]

- **initial value**
  [0]
Reduce operations

Three fundamental pieces:

function

\[ f(x,y) = x+y \]

iterable

\[ \begin{align*}
2 & \quad 3 & \quad 5 & \quad 8 & \quad 1 & \quad 1 & \quad 7 \\
\end{align*} \]

initial value

\[ 0 \]

accumulator

\[ 0 \]

Python initializes an accumulator with the given initial value. Think of the accumulator as a “running total”.

Reduce operations

Three fundamental pieces:

- **function**
  \[ f(x,y) = x+y \]

- **iterable**
  \begin{align*}
  2 & \quad 3 & \quad 5 & \quad 8 & \quad 1 & \quad 1 & \quad 7
  \end{align*}

- **initial value**
  \begin{align*}
  0
  \end{align*}

Now, Python repeatedly updates the accumulator, with

\[
\text{accumulator} = f(\text{accumulator}, y)
\]

where \( y \) traverses the sequence
Reduce operations

Three fundamental pieces:

function

\[ f(x,y) = x+y \]

iterable

2 3 5 8 1 1 7

initial value

0

Now, Python repeatedly updates the accumulator, with
\[ \text{accumulator} = f(\text{accumulator}, y) \]
where \( y \) traverses the sequence
Reduce operations

Three fundamental pieces:

- **function**
  \[ f(x,y) = x + y \]

- **iterable**
  \[ 2 \quad 3 \quad 5 \quad 8 \quad 1 \quad 1 \quad 7 \]

- **initial value**
  \[ 0 \]

Now, Python repeatedly updates the accumulator, with

\[ \text{accumulator} = f(\text{accumulator}, y) \]

where \( y \) traverses the sequence.
Reduce operations

Three fundamental pieces:

- Function: \( f(x, y) = x + y \)
- Iterable: \( 2, 3, 5, 8, 1, 1, 7 \)
- Initial value: \( 0 \)

Now, Python repeatedly updates the accumulator, with accumulator = f(accumulator, y) where \( y \) traverses the sequence.
Reduce operations

Three fundamental pieces:

function

\[ f(x,y) = x+y \]

iterable

\[ 2 \quad 3 \quad 5 \quad 8 \quad 1 \quad 1 \quad 7 \]

initial value

\[ 0 \]

Now, Python repeatedly updates the accumulator, with

\[ \text{accumulator} = f(\text{accumulator}, y) \]

where \( y \) traverses the sequence.
Reduce operations

Three fundamental pieces:

- Function: \( f(x,y) = x+y \)
- Iterable: \( [2, 3, 5, 8, 1, 1, 7] \)
- Initial value: 0

Now, Python repeatedly updates the accumulator, with
\[ \text{accumulator} = f(\text{accumulator}, y) \]
where \( y \) traverses the sequence.
Reduce operations

Three fundamental pieces:

- **function**: $f(x, y) = x + y$
- **iterable**: $[2, 3, 5, 8, 1, 1, 7]$
- **initial value**: $0$

Now, Python repeatedly updates the accumulator, with $\text{accumulator} = f(\text{accumulator}, y)$ where $y$ traverses the sequence.
Reduce operations

Three fundamental pieces:

\[ f(x, y) = x + y \]

Now, Python repeatedly updates the accumulator, with

\[ \text{accumulator} = f(\text{accumulator}, y) \]

where \( y \) traverses the sequence
Reduce operations

Three fundamental pieces:

function

\[ f(x,y) = x+y \]

iterable

2  3  5  8  1  1  7

initial value

0

accumulator

10

...and so on.
Reduce operations

Three fundamental pieces:

- **Function**
  \[ f(x, y) = x + y \]

- **Iterable**
  \[ 2 \quad 3 \quad 5 \quad 8 \quad 1 \quad 1 \quad 7 \]

- **Initial Value**
  \[ 0 \]

Once Python gets a `StopIteration` error indicating that the iterator has no more elements, it returns the value in the accumulator.
Reduce operations

Three fundamental pieces:

<table>
<thead>
<tr>
<th>function</th>
<th>iterable</th>
<th>initial value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x,y) = x+y )</td>
<td>0 2 3 5 8 1 1 7</td>
<td>0</td>
</tr>
</tbody>
</table>

If the initial value isn’t supplied, Python initializes the accumulator as \( \text{acc} = f(x, y) \) where \( x \) and \( y \) are the first two elements of the iterator. If the iterator is length 1, it just returns that element. All told, it’s best to always specify the initial value, except in very simple cases (like these slides).
Reduce operations

Three fundamental pieces:

function

\[ f(x,y) = x + y \]

iterable

\[ 2 \ 3 \ 5 \ 8 \ 1 \ 1 \ 7 \]

initial value

\[ 0 \]

**Warning:** if the iterator supplied to `reduce` is empty, then we really do need the initial value!

```
1  functools.reduce(lambda x,y: x+y, [])
```

```
TypeError: reduce() of empty sequence with no initial value
```
Reduce in Python

`reduce` is not included as a built-in function in Python, unlike `map` and `filter`.

Because developers felt that `reduce` is not “Pythonic”

The argument is that `reduce` operations can always be written as a for-loop:

```
import functools
functools.reduce(lambda x,y: x+y, range(10))
```

```
acc = 0
for i in range(10):
    acc += i
acc
```
Reduce in Python

reduce is not included as a built-in function in Python, unlike map and filter
Because developers felt that reduce is not “Pythonic”

The argument is that reduce operations can always be written as a for-loop:

```
1 import functools
2 functools.reduce(lambda x,y: x+y, range(10))
```

```
1 acc = 0
2 for i in range(10):
3     acc += i
4 acc
```

This criticism is mostly correct, but we’ll see later in the course when we cover MapReduce that there are cases where we really do want a proper reduce function.
Reduce in Python

All of the standard reduce-like functions are easily reimplemented with reduce statements, like this example, with \texttt{max}. Note the use of Python’s in-line conditional statement.

More often, one has to implement the pairwise function. For example, here we have implemented a function for entrywise addition of tuples.

Note: there are “more functional” ways to do this. Since tuples are themselves iterable, we could write a clever “function of functions” to do this more gracefully. More on this soon.
Quantifiers over iterables: `any()` and `all()`

- **`any()`**
  - Takes an iterable as its input and returns `True` if and only if one or more elements is `True`.
  - Example:
    ```python
    any([False,True,False])
    True
    any((0,'',0.0))
    False
    any(map(is_even,fibo))
    False
    ```
  - Reminder: `0`, `0.0`, empty string, empty list, etc all evaluate to `False`. Just about everything else evaluates to `True`.

- **`all()`**
  - Takes an iterable as its input and returns `True` if and only if all elements are `True`.
  - Example:
    ```python
    all([(1,0),1,'cat'])
    True
    ```
Quantifiers over iterables: \texttt{any()} and \texttt{all()}

Here’s a nice example of why functional programming is useful. Complicated functions become elegant one-liners!

```python
1  def is_prime(n):
2      return not any((n\%x==0 for x in range(2,n)))
3  is_prime(8675309)

True
```

Of course, sometimes that elegance comes at the cost of efficiency. In this example, we’re failing to use a speedup that would be gained from using, e.g., the sieve of Eratosthenes and stopping checking above $\sqrt{n}$.

```python
1  is_prime(8675310)

False
```

any and all are lazy

As soon as `any` finds a `True` element, it returns `True`. As soon as `all` finds a `False` element, it returns `False`. This is a simpler (i.e., less general) notion of laziness than lazy evaluation, but the underlying motivation is the same. Do as little work as is necessary to get your answer!
Related: `itertools.accumulate`

`itertools.accumulate` performs a reduce operation, but it returns an iterator over the partial “sums” of its argument. Returns an empty iterator if argument is empty.

```python
1 itertools.accumulate(range(1,10), lambda x,y:x+y)
<itertools.accumulate at 0x10a6aa348>

1 list(itertools.accumulate(range(1,10), lambda x,y:x+y))
[1, 3, 6, 10, 15, 21, 28, 36, 45]

1 list(itertools.accumulate([(1,2),(1,3),(2,5),(3,7),(5,11)], tuple_add))
[(1, 2), (2, 5), (4, 10), (7, 17), (12, 28)]
```

I put “sums” in quotes above, because of course the function need not be addition. The point is that we get an iterator over the values of the accumulator at each step of the reduce operation.
Working with iterators: `itertools`

- `itertools.count(x, y)` returns an infinite iterator of numbers starting at $x$ and proceeding in increments of $y$.

- `itertools.accumulate(t)` returns an iterator of partial sums of $t$. Or partial “sums” if we specify a different function.

- `itertools.filterfalse(t)` is like the opposite of `filter`.

- `itertools.starmap` similar to `map`, but applies multi-argument function to tuples. Name is reference to the `*args` notation.

```python
import itertools
def is_even(n):
    return n % 2 == 0
def fibo(n):
    if n <= 1:
        return n
    else:
        return fibo(n-1) + fibo(n-2)

sevens = itertools.count(7, 7)
[next(sevens) for x in range(10)]
[7, 14, 21, 28, 35, 42, 49, 56, 63, 70]

list(itertools.accumulate(range(10)))
[0, 1, 3, 6, 10, 15, 21, 28, 36, 45]

list(itertools.accumulate(range(1,10), max))
[1, 2, 3, 4, 5, 6, 7, 8, 9]

list(itertools.filterfalse(is_even, fibo))
[1, 1, 3, 5, 13]

list(itertools.starmap(poly, [[(1,1), (1,2), (2,1), (3,4)]]))
[-3, -3, -5, -1]
```

[https://docs.python.org/3/library/itertools.html#module-itertools](https://docs.python.org/3/library/itertools.html#module-itertools)
More `itertools`: combinations

```python
1 list(itertools.combinations([1,2,3,4], 2))
[(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)]

1 list(itertools.permutations([1,2,3], 2))
[(1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2)]

1 list(itertools.combinations_withReplacement([1,2,3,4], 2))
[(1, 1),
 (1, 2),
 (1, 3),
 (1, 4),
 (2, 2),
 (2, 3),
 (2, 4),
 (3, 3),
 (3, 4),
 (4, 4)]
```

`itertools` also includes some combinatorial functions that can be useful on occasion.
Aside: Python `operator` module

It’s awfully annoying to have to write `lambda x,y: x+y` all the time.

```python
1. functools.reduce(lambda x, y: x*y, range(1,10))
```

362880

```python
1. functools.reduce(*, range(1,10))
```

File "<ipython-input-90-46140374121>", line 1
```
```
functools.reduce(*, range(1,10)))
```

SyntaxError: invalid syntax

Here is what we’d like to write, but of course it’s a syntax error.

`operator.mul` gives us `*`, but as a function, just as though we wrote a lambda expression.

`import operator`
```
2. functools.reduce(operator.mul, range(1,10))
```

362880

`operator` includes many other functions:

Math: `add()`, `sub()`, `mul()`, `abs()`, etc.
Logic: `not_()`, `truth()`.
Bitwise: `and_()`, `or_()`, `invert()`.
Comparison: `eq()`, `ne()`, `lt()`, `le()`, etc.
Identity: `is_()`, `is_not()`.

[https://docs.python.org/3/library/operator.html#module-operator](https://docs.python.org/3/library/operator.html#module-operator)
More functional patterns: `functools`

`functools` module provides a number of functional programming constructions.

`functools.partial` takes a function and a set of arguments to pass to the function. Returns a function with some of its arguments “fixed”.

```python
import functools
pow2 = functools.partial(math.pow, 2)
pow2

functools.partial(<built-in function pow>, 2)
```

So in this case, it’s like we got a new function, `pow2(x) == math.pow(2, x)`.

```python
list(map(pow2, range(10)))
[1.0, 2.0, 4.0, 8.0, 16.0, 32.0, 64.0, 128.0, 256.0, 512.0]
```

`functools.partial` also lets us pass keyword arguments.

```python
def my_pow(x=1, y=1):
    return math.pow(x, y)
my_square = functools.partial(my_pow, y=2)
list(map(my_square, range(10)))
[0.0, 1.0, 4.0, 9.0, 16.0, 25.0, 36.0, 49.0, 64.0, 81.0]
```
Higher-order functions and currying

`functools.partial` takes a function (and other stuff), returns a function
Called a higher-order function

In most other languages, Python’s `functools.partial` is called currying

Currying is named after logician Haskell Curry

https://en.wikipedia.org/wiki/Currying

```
1 f = lambda x,y,z : x*y*z
2 curry1 = functools.partial(f,2)
3 curry2 = functools.partial(curry1,3)
4 curry2(4)
```

Curry1 takes two arguments, returns their product times 2.

```
1 par = functools.partial(f,2,3)
2 par(4)
```

Curry2 takes one argument `z`, returns `2*3*z` (reminder: partial fills positional arguments in order).

Equivalently, just pass both arguments in one call to partial.
Pure functions, again

Recall that a **pure function** is a function that did not have any side effects.

Pure functions are especially important in functional programming. A pure function is really a function (in the mathematical sense). Given the same input, it always produces the same output (And doesn’t change the state of our program!)

```python
a = 0
def increment_mod():
    global a
    a += 1
def increment_pure(x):
    return x+1
```

This function is a modifier. It has side effects.

This is a pure function.
Pure functions, again

Recall that a **pure function** was a function that did not have any side effects.

Pure functions are especially important in functional programming.

A pure function is really a function (in the mathematical sense).

Given the same input, it always produces the same output.

(And doesn’t change the state of our program!)

---

<table>
<thead>
<tr>
<th>1</th>
<th><code>a = 0</code></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td><code>def increment_mod():</code></td>
</tr>
<tr>
<td>3</td>
<td><code>global a</code></td>
</tr>
<tr>
<td>4</td>
<td><code>a += 1</code></td>
</tr>
<tr>
<td>5</td>
<td><code>def increment_pure(x):</code></td>
</tr>
<tr>
<td>6</td>
<td><code>return x+1</code></td>
</tr>
</tbody>
</table>

---

Pure functions are also crucial to having **immutable data**. Think about processing the observations in a data set. We don’t want to change the original data file in the process of our analysis! We want to be able to write a pipeline, in which we pass data from one function to another, producing a transformed version of the data at each step.
Pure functions and higher-order functions

Pure functions arise frequently in map/reduce frameworks

A good example of a higher-order function: `compose` takes some functions and produces a new function.

```python
def compose(*funcs):
    '''Return a new function that is the composition of the argument functions.'''
    def inner(data, funcs=funcs):
        result = data
        for f in reversed(funcs):
            result = f(result)
        return result

f = lambda x: x**2
g = lambda x: x + 1
# compose(g,f) == g(f(x)) == x**2 + 1
list(map(compose(g,f), range(10)))
```

Returning a function is okay, because Python has first-class functions.

You can see why we prefer pure functions for this. If `f` and/or `g` had side effects, this would be a big mess!

Example credit: D. Mertz, Functional Programming in Python
Of course, I'm exaggerating the complexity of this object here, but this really is what object-oriented code ends up looking like in the wild.

Contrast that with the simplicity of this functional version of the same letter-counting operation.
Why use functional programming?

Some problems are especially well-suited to this paradigm

Example: quicksort

```python
def quicksort(t):
    if len(t) <= 1:
        return t # list is already sorted.
    else:
        pivot = t[0]
        return (quicksort([x for x in t if x < pivot]) +
                [x for x in t if x == pivot] +
                quicksort([x for x in t if x > pivot]))

quicksort([3, 5, 4, 2, 1, 6, 5, 7, 4, 0, 0, 2, 4])
```

See the quicksort Wikipedia page for examples of what this looks like when written in a non-functional style.

https://en.wikipedia.org/wiki/Quicksort
A note on recursion in Python: tail call optimization

“Tail call optimisation is a programming language feature. Each time a function recurses, a new stack frame is created. A stack frame is used to store the arguments and local values for the current function invocation. If a function recurses a large number of times, it is possible for the interpreter or compiler to run out of memory. Languages with tail call optimisation reuse the same stack frame for their entire sequence of recursive calls. Languages like Python that do not have tail call optimisation generally limit the number of times a function may recurse to some number in the thousands.”

Python doesn’t have tail call recursion, so some functional programing patterns simply aren’t well-suited if we may encounter many thousands of layers of recursion. Recall our memoized function for computing the Fibonacci numbers.
Declarative Programming

Describe what the program \textbf{should do}, rather than how it does it
Implementation details are left up to the language as much as possible

Contrast with \textbf{imperative/procedural programming}
Sequence of statements describes \textbf{how} program should proceed
Most programming you have done in the past is procedural
Program consists of subroutines that get called, change state of program

Don’t worry too much about these distinctions. Most languages are a mix of paradigms, and no single approach is a silver bullet.
\textbf{Different applications call for different programming paradigms.}
Congratulations! You know enough functional programming to get the joke in this xkcd comic!

*Alt-text:* Functional programming combines the flexibility and power of abstract mathematics with the intuitive clarity of abstract mathematics.

https://xkcd.com/1270/