

# STAT679

# Computing for Data Science and Statistics

Lecture 9: `numpy` and `scipy`

Some examples adapted from A. Tewari @ UMichigan

# Numerical computing in Python: `numpy`

One of a few increasingly-popular, free competitors to MATLAB

Numpy quickstart guide: <https://numpy.org/doc/stable/user/quickstart.html>

For MATLAB fans:

<https://docs.scipy.org/doc/numpy-dev/user/numpy-for-matlab-users.html>

Closely related package `scipy` is for optimization

See <https://docs.scipy.org/doc/>

# Installing packages

So far, we have only used built-in modules

But there are many modules/packages that do not come preinstalled

Ways to install packages:

At the `conda` prompt or in terminal: `conda install numpy`

<https://conda.io/docs/user-guide/tasks/manage-pkgs.html>

Using `pip` (recommended): `pip install numpy`

<https://pip.pypa.io/en/stable/>

Using UNIX/Linux package manager (not recommended)

From source (not recommended)



# numpy data types

`import ... as ...` lets us import a package and give it a shorter name.

```
1 import numpy as np
2
3 x = np.float32(3.1415)
4 type(x)
```

`numpy.float32`

Five basic numerical data types:

boolean (`bool`)

integer (`int`)

unsigned integer (`uint`)

floating point (`float`)

complex (`complex`)

Note that this is not the same as a Python `int`.

```
1 x
```

3.1415

```
1 x = np.int(8675309)
```

```
2 x
```

8675309

Many more complicated data types are available

e.g., each of the numerical types can vary in how many bits it uses

<https://docs.scipy.org/doc/numpy/user/basics.types.html>

# numpy data types

```
1 x = np.float64(3.1415)
2 x
```

3.1415

```
1 y = np.float32(3.1415)
2 type(y)
```

numpy.float32

```
1 x==y
```

False

```
1 x==np.float64(y)
```

False

```
1 x = np.int_(8675309)
2 type(x)
```

numpy.int64

32-bit and 64-bit representations are distinct!

Data type followed by underscore uses the default number of bits. This default varies by system.

As a rule, it's best never to check for equality of floats. Instead, check whether they are within some error tolerance of one another.

`numpy.array`: `numpy`'s version of Python array (i.e., list)

Can be created from a Python list...

```
1 np.array([1, 2, 3], dtype='uint')  
array([1, 2, 3], dtype=uint64)
```

...by “shaping” an array...

```
1 np.zeros((2,3))  
array([[ 0.,  0.,  0.],  
       [ 0.,  0.,  0.]])
```

`np.zeros` and `np.ones` generate arrays of 0s or 1s, respectively. Shape parameter (2,3) means to create a 2-D array with two rows and three columns.

...by “ranges”...

```
1 np.arange(2, 3, 0.1, dtype='float')  
array([ 2. ,  2.1,  2.2,  2.3,  2.4,  2.5,  2.6,  2.7,  2.8,  2.9])
```

...or reading directly from a file

see <https://docs.scipy.org/doc/numpy/user/basics.creation.html>

# numpy allows arrays of arbitrary dimension (tensors)

1-dimensional arrays:

```
1 x = np.arange(12) # x=[1,2,...,12]
2 x
```

```
array([ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9, 10, 11])
```

2-dimensional arrays (matrices):

```
1 x.shape = (3,4) # now x is a 3-by-4 matrix
2 x # observe that shape fills the new matrix by row.
```

```
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

3-dimensional arrays ("3-tensor"):

```
1 x.shape = (2,3,2)
2 x # now x is a 2-by-3-by-2 "cube" of numbers
```

```
array([[[ 0,  1],
        [ 2,  3],
        [ 4,  5]],
       [[ 6,  7],
        [ 8,  9],
        [10, 11]])])
```



# numpy allows arrays of arbitrary dimension (tensors)

1-dimensional arrays:

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2 x
```

```
array([ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9, 10, 11])
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array([[ 0,  1,  2,  3],
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```

3-dimensional arrays ("3-tensor"):

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1 x.shape = (2,3,2)
2 x # now x is a 2-by-3-by-2 "cube" of numbers
```

```
array([[[ 0,  1],
        [ 2,  3],
        [ 4,  5]],
       [[ 6,  7],
        [ 8,  9],
        [10, 11]])])
```

Every numpy array has a `shape` attribute specifying its dimensions. For example, an array with shape (3,4) has three rows and four columns. An array with shape (2,3,2) is a 2-by-3-by-2 "box" of numbers.

Think of the shape of an array as specifying how many indices we need to pick out an entry of the array. For example, to pick out a number from a 3-by-4 matrix, we must specify a row and a column.

# More on `numpy.arange` creation

`np.arange(x)`: array version of Python's `range(x)`, like `[0, 1, 2, ..., x-1]`

`np.arange(x, y)`: array version of `range(x, y)`, like `[x, x+1, ..., y-1]`

`np.arange(x, y, z)`: array of elements `[x, y)` in `z`-size increments.

```
1 np.arange(10)
```

```
array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])
```

```
1 np.arange(5, 10)
```

```
array([5, 6, 7, 8, 9])
```

```
1 np.arange(0, 1, 0.1)
```

```
array([ 0. ,  0.1,  0.2,  0.3,  0.4,  0.5,  0.6,  0.7,  0.8,  0.9])
```

## More on `numpy.arange` creation

`np.arange(x)`: array version of Python's `range(x)`, like `[0, 1, 2, ..., x-1]`

`np.arange(x, y)`: array version of `range(x, y)`, like `[x, x+1, ..., y-1]`

`np.arange(x, y, z)`: array of elements `[x, y)` in `z`-size increments.

Related useful functions, that give better/clearer control of start/endpoints and allow for multidimensional arrays:

<https://docs.scipy.org/doc/numpy/reference/generated/numpy.linspace.html>

<https://docs.scipy.org/doc/numpy/reference/generated/numpy.ogrid.html>

<https://docs.scipy.org/doc/numpy/reference/generated/numpy.mgrid.html>

# numpy array indexing is highly expressive

```
1 x = np.arange(10)
2 x[2:5]
```

```
array([2, 3, 4])
```

```
1 x[:-7]
```

```
array([0, 1, 2])
```

```
1 x[1:7:2]
```

```
array([1, 3, 5])
```

```
1 x[::2]
```

```
array([0, 2, 4, 6, 8])
```

Slices, strides, indexing from the end, etc.  
Just like with Python lists.

Not very relevant to us right now...

...but this will come up again in a few weeks when we cover TensorFlow

# More array indexing

```
1 x = np.reshape(np.arange(1,13), (3,4))
2 x
```

```
array([[ 1,  2,  3,  4],
       [ 5,  6,  7,  8],
       [ 9, 10, 11, 12]])
```

If we specify fewer than the number of indices, `numpy` assumes we mean `:` in the remaining indices.

```
1 x[1]
```

```
array([5, 6, 7, 8])
```

**Warning:** if you're used to MATLAB or R, this behavior will seem weird to you.

```
1 x[:,(1,3)]
```

```
array([[ 2,  4],
       [ 6,  8],
       [10, 12]])
```

**From the documentation:** When the index consists of as many integer arrays as the array being indexed has dimensions, the indexing is straight forward, but different from slicing. Advanced indexes always are [broadcast](#) and iterated as *one*.

```
1 x[(0,2),(1,3)]
```

```
array([ 2, 12])
```

<https://docs.scipy.org/doc/numpy/reference/arrays.indexing.html#integer-array-indexing>

# More array indexing

Numpy allows MATLAB/R-like indexing by Booleans

```
1 x = np.arange(10)
2 x[x>7]
```

```
array([8, 9])
```

```
1 x[(x>7) or (x<2)]
```

```
-----
ValueError                                Traceback (most recent call last)
<ipython-input-373-6b519499a034> in <module>()
----> 1 x[(x>7) or (x<2)]
```

```
ValueError: The truth value of an array with more than one element is ambiguous. Use a.any() or a.all()
```

Believe it or not, this error is by design! The designers of `numpy` were concerned about ambiguities in Boolean vector operations. In essence, should `(x>7) or (x<2)` be a vector of Booleans or a single Boolean?

# Boolean operations: `np.any()`, `np.all()`

```
1 x = np.arange(10)
2 np.all(x>7)
```

False

Just like the `any` and `all` functions in Python proper.

```
1 np.any(x>7)
```

True

```
1 np.any([x>7,x<2])
```

True

```
1 np.any([x>7,x<2], axis=1)
```

```
array([ True,  True], dtype=bool)
```

```
1 np.any([x>7,x<2], axis=0)
```

```
array([ True,  True, False, False, False, False, False, False,  True,  True], dtype=bool)
```

`axis` argument picks which axis along which to perform the Boolean operation. If left unspecified, it treats the array as a single vector.

Setting `axis` to be the first (i.e., 0-th) axis yields the entrywise behavior we wanted.

# Boolean operations: `np.logical_and()`

`numpy` also has built-in Boolean vector operations, which are simpler/clearer at the cost of the expressiveness of `np.any()`, `np.all()`.

```
1 x = np.arange(10)
2 x[np.logical_and(x>3,x<7)]
```

```
array([4, 5, 6])
```

```
1 np.logical_or(x<3,x>7)
```

```
array([ True,  True,  True, False, False, False, False, False,  True,  True], dtype=bool)
```

```
1 x[np.logical_xor(x>3,x<7)]
```

```
array([0, 1, 2, 3, 7, 8, 9])
```

```
1 x[np.logical_not(x>3)]
```

```
array([0, 1, 2, 3])
```

This is an example of a numpy “universal function” (ufunc), which we’ll discuss more in a few slides.



# Random numbers in numpy

`np.random` contains methods for generating random numbers

```
1 np.random.random((2,3))
```

```
array([[ 0.61420793,  0.46363275,  0.22880783],  
       [ 0.24268979,  0.13462754,  0.6026283 ]])
```

```
1 np.random.normal(0,1,20)
```

```
array([ 1.31323138,  0.76807767,  1.92180038, -0.34121468,  0.72572401,  
       1.0273551 , -0.78435871,  0.42732636,  1.05947171,  0.23042635,  
       0.3951938 ,  0.3595342 ,  0.14710555,  0.42279814,  0.84381846,  
       1.06495165, -1.51074354, -0.16419861,  2.89275956, -1.18501386])
```

```
1 np.random.uniform(0,1,(2,4))
```

```
array([[ 0.08399452,  0.03934797,  0.3603464 ,  0.66361677],  
       [ 0.33499095,  0.29427732,  0.14963153,  0.87892145]])
```

Lots more distributions:

<https://docs.scipy.org/doc/numpy/reference/routines.random.html#distributions>

# `np.random.choice()`: random samples from data

`np.random.choice(x, [size, replace, p])`

Generates a sample of `size` elements from the array `x`, drawn with (`replace=True`) or without (`replace=False`) replacement, with element probabilities given by vector `p`.

```
1 x = np.arange(1,11)
2 for i in range(5):
3     print np.random.choice(x,5,False,x/float(sum(x)))
```

```
[ 1  5 10  7  6]
[ 8  5  9  2  6]
[ 9  6  3  8 10]
[ 7  9 10  5  6]
[ 8  5  6  9  1]
```

# shuffle() VS permutation()

`np.random.shuffle(x)`

randomly permutes entries of `x` in place  
so `x` itself is changed by this operation!

`np.random.permutation(x)`

returns a random permutation of `x`  
and `x` remains unchanged.

Compare with the Python `list.sort()`  
and `sorted()` functions.

```
1 x = np.arange(10)
2 print x
```

```
[0 1 2 3 4 5 6 7 8 9]
```

```
1 np.random.shuffle(x)
2 print x # x is different, now.
```

```
[1 5 0 3 2 7 6 8 9 4]
```

```
1 print np.random.permutation(x)
```

```
[5 2 8 7 0 3 9 6 1 4]
```

```
1 print x # x is unchanged by permutation()
```

```
[1 5 0 3 2 7 6 8 9 4]
```

# Statistics in `numpy`

`numpy` implements all the standard statistics functions you've come to expect

```
1 x = np.random.normal(0,1,100)
2 np.mean(x), np.median(x), np.std(x)
```

```
(-0.062724875643358866, -0.05261873350441526, 1.0556291754262765)
```

```
1 np.min(x), np.max(x), np.ptp(x) # ptp gets max-min
```

```
(-3.1029568746428113, 1.9628924810049164, 5.0658493556477282)
```

```
1 np.std(x), np.var(x)
```

```
(1.0556291754262765, 1.1143529560111607)
```

# Statistics in `numpy`

NaN is short for “not a number”. NaNs typically arise either because of improper mathematical operations (e.g., dividing by zero) or to represent missing data.

`Numpy` deals with NaNs more gracefully than `MATLAB/R`:

```
1 x[5] = np.nan
2 np.mean(x)
```

nan

```
1 np.nanmin(x), np.nanmax(x), np.nanstd(x), np.nanvar(x)
```

```
(-3.1029568746428113,
 1.9628924810049164,
 1.0439479158102707,
 1.0898272509246081)
```

`nanmin`, `nanvar`, etc compute function after dropping NaNs.

For more statistical functions, see:

<https://docs.scipy.org/doc/numpy-1.8.1/reference/routines.statistics.html>

# Probability and statistics in `scipy`

`scipy` is a distinct Python package, part of the `numpy` ecosystem.

(Almost) all the distributions you could possibly ever want:

<https://docs.scipy.org/doc/scipy/reference/stats.html#continuous-distributions>

<https://docs.scipy.org/doc/scipy/reference/stats.html#multivariate-distributions>

<https://docs.scipy.org/doc/scipy/reference/stats.html#discrete-distributions>

More statistical functions (moments, kurtosis, statistical tests):

<https://docs.scipy.org/doc/scipy/reference/stats.html#statistical-functions>

```
1 import scipy.stats
2 x = np.random.normal(0, 1, 20)
3 scipy.stats.kstest(x, 'norm')
```

Second argument is the name of a distribution in `scipy.stats`

```
KstestResult(statistic=0.23182037538316391, pvalue=0.19897055187485568)
```

[Kolmogorov-Smirnov test](#)

# Matrix-vector operations in `numpy`

```
1 A = np.reshape(np.arange(1,13), (3,4))
2 x = np.ones(4)
3 A*x
```

```
array([[ 1.,  2.,  3.,  4.],
       [ 5.,  6.,  7.,  8.],
       [ 9., 10., 11., 12.]])
```

Trying to multiply two arrays, and you get **broadcast** behavior, *not* a matrix-vector product.

```
1 y = np.ones(3)
2 A*y
```

```
-----
ValueError                                Traceback (most recent call last)
<ipython-input-83-86c92ad89b88> in <module>()
      1 y = np.ones(3)
----> 2 A*y
```

```
ValueError: operands could not be broadcast together with shapes (3,4) (3,)
```

```
1 np.reshape(y, (3,1))*A
```

```
array([[ 1.,  2.,  3.,  4.],
       [ 5.,  6.,  7.,  8.],
       [ 9., 10., 11., 12.]])
```

Broadcast multiplication still requires that dimensions agree and all that.

# Matrix-vector operations in `numpy`

```
1 A = np.matrix(np.reshape(np.arange(1,13),(3,4)))
```

```
2 A
```

```
matrix([[ 1,  2,  3,  4],  
        [ 5,  6,  7,  8],  
        [ 9, 10, 11, 12]])
```

Create a numpy matrix from a numpy array. We can also create matrices from strings with MATLAB-like syntax. See documentation.

```
1 x = np.ones((4,1))
```

```
2 A*x
```

```
matrix([[10.],  
        [26.],  
        [42.]])
```

Now matrix-vector and vector-matrix multiplication work as we want.

```
1 y = np.ones((1,3))
```

```
2 y*A
```

```
matrix([[15., 18., 21., 24.]])
```

Numpy matrices support a whole bunch of useful methods. See documentation: <https://docs.scipy.org/doc/numpy/reference/generated/numpy.matrix.html>



# numpy/scipy universal functions (ufuncs)

From the documentation:

A universal function (or ufunc for short) is a function that operates on ndarrays in an element-by-element fashion, supporting array broadcasting, type casting, and several other standard features. That is, a ufunc is a “vectorized” wrapper for a function that takes a fixed number of scalar inputs and produces a fixed number of scalar outputs.

<https://docs.scipy.org/doc/numpy/reference/ufuncs.html>

So ufuncs are vectorized operations, just like in R and MATLAB

# ufuncs in action

List comprehensions are great, but they're not well-suited to numerical computing

```
1 x = range(10)
2 x**2
```

```
-----
TypeError                                 Traceback (most recent call last)
<ipython-input-466-84f8296342ab> in <module>()
      1 x = range(10)
----> 2 x**2
```

```
TypeError: unsupported operand type(s) for ** or pow(): 'list' and 'int'
```

```
1 [x**2 for x in np.arange(10)]
```

```
[0, 1, 4, 9, 16, 25, 36, 49, 64, 81]
```

```
1 x = np.arange(10)
2 x**2
```

```
array([ 0,  1,  4,  9, 16, 25, 36, 49, 64, 81])
```

Unlike Python lists, `numpy` arrays support vectorized operations.

# Sorting

```
1 charray = np.array([c for c in 'Go Badgers']).reshape((2,5))
2 print(charray)
```

```
[[ 'G' 'o' ' ' 'B' 'a' ]
 [ 'd' 'g' 'e' 'r' 's' ]]
```

ASCII rears its head-- capital letters are “smaller” than all lower-case by default.

```
1 np.sort(charray)
```

```
array([[ ' ' 'B' 'G' 'a' 'o' ],
       [ 'd' 'e' 'g' 'r' 's' ]], dtype='<U1')
```

```
1 np.sort(charray, axis=1)
```

```
array([[ ' ' 'B' 'G' 'a' 'o' ],
       [ 'd' 'e' 'g' 'r' 's' ]], dtype='<U1')
```

Sorting is along the “last” axis by default. Note contrast with `np.any()`. To treat the array as a single vector, `axis` must be set to `None`.

```
1 np.sort(charray, axis=0)
```

```
array([[ 'G' 'g' ' ' 'B' 'a' ],
       [ 'd' 'o' 'e' 'r' 's' ]], dtype='<U1')
```

```
1 np.sort(charray, axis=None)
```

```
array([ ' ' 'B' 'G' 'a' 'd' 'e' 'g' 'o' 'r' 's' ], dtype='<U1')
```

```
1 print(charray)
```

```
[[ 'G' 'o' ' ' 'B' 'a' ]
 [ 'd' 'g' 'e' 'r' 's' ]]
```

Original array is unchanged by use of `np.sort()`, like Python’s built-in `sorted()`

# A cautionary note

`numpy/scipy` have several similarly-named functions with different behaviors!

Example: `np.amax`, `np.ndarray.max`, `np.maximum`

The best way to avoid these confusions is to

- 1) Read the documentation carefully
- 2) Test your code!