(1) Find all integers $a$, $b$, and $c$ for which
\[(x - a)(x - 10) + 1 = (x + b)(x + c)\] for all $x$.

(2) Let $A$ be a positive real number. What are the possible values of $\sum_{j=0}^{\infty} x_j^2$, given that $x_0, x_1, \ldots$ are positive numbers for which $\sum_{j=0}^{\infty} x_j = A$.

(3) Prove that there exist infinitely many integers $n$ such that $n, n + 1, n + 2$ are each the sum of the squares of two integers. (For example, $0 = 0^2 + 0^2$, $1 = 0^2 + 1^2$, $2 = 1^2 + 1^2$.)

(4) For any two integers $m$ and $n$ with $0 \leq m \leq n$, numbers $d(m, n)$ are defined by
\[d(n, 0) = d(n, n) = 1\] for all $n \geq 0$

and
\[m \cdot d(n, m) = m \cdot d(n - 1, m) + (2n - m) \cdot d(n - 1, m - 1)\] for $0 < m < n$.

Prove that all of the $d(n, m)$ are integers.