Brain Network from Sparse and Topological point of view

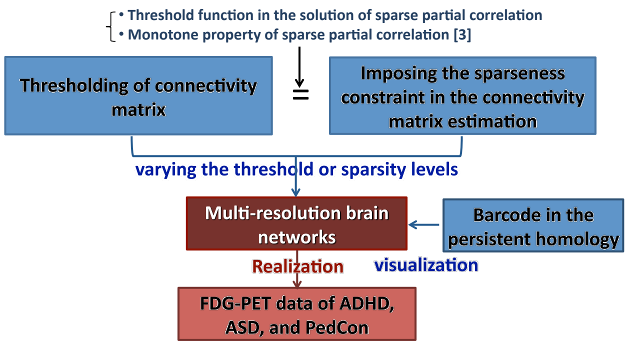
Hyekyoung Lee[1], Moo K. Chung[1,2], Hyejin Kang[1], Boong-Nyun Kim[1], Dong Soo Lee[1]

[1]Seoul National University, Seoul, Republic of Korea, [2]University of Wisconsin, Madison, WI

Introduction

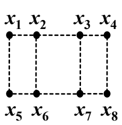
- The sparse brain network is usually obtained by two different ways : thresholding connectivity matrix and imposing the sparseness constraint in the connectivity matrix estimation [1]. However, it is not yet clear what threshold or sparseness level is best in determining the connectivity structure of the brain.
- In this work, we show the equivalence between sparseness and threshold, and propose to look at the topological changes of network by varying the threshold/sparseness, without using the fixed threshold/sparseness. For visualization and quantification, we used the concept of barcodes in the persistent homology [2]
- As an illustration, we apply the proposed method to constructing the FDG-PET based functional brain networks out of 24 attention-deficit hyperactivity disorder (ADHD) children, 26 autism-spectrum disorder (ASD) children and 11 pediatric control (PedCon) subjects.

Outline



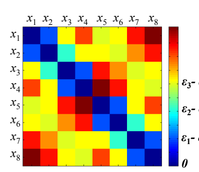
Persistent Homology

Given point cloud data X & their metric $d(x_i, x_j)$,



$X = \{x_1, x_2, \ldots, x_8\}$

$$\begin{cases} d(x_1, x_2) = d(x_3, x_4) = \ldots = \varepsilon_1 - \delta, \\ d(x_2, x_3) = d(x_6, x_7) = \varepsilon_2 - \delta, \\ d(x_1, x_5) = d(x_2, x_6) = \ldots = \varepsilon_3 - \delta \end{cases}$$



Connectivity matrix $d(x_i, x_j)$

- **Rips complex, $\mathrm{Rips}(X, \varepsilon)$**

  approximate the topology of the point cloud data by connecting two point cloud data, x_i and x_j if $d(x_i, x_j) < \varepsilon$

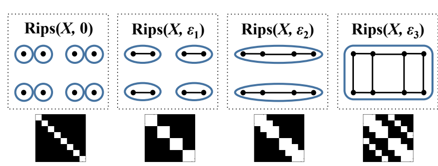
- **Rips filtration**

  the sequence of Rips complexes satisfying the persistent property such as $\mathrm{Rips}(X, 0) \subseteq \mathrm{Rips}(X, \varepsilon_1) \subseteq \mathrm{Rips}(X, \varepsilon_2) \subseteq \ldots \subseteq \mathrm{Rips}(X, \varepsilon_n)$ for $0 \le \varepsilon_1 \le \varepsilon_2 \le \ldots \le \varepsilon_n$

Rips filtration

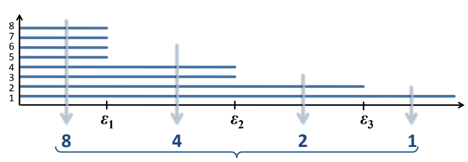
&

its adjacency matrices



- **Barcode**

  visualize the changes of the connected components during the filtration



Number of connected components = the 0-th Betti number

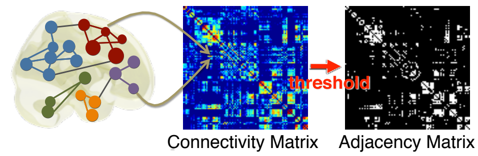
References

[1] Lee, H., et. al. (2011), TMI, vol. 30.

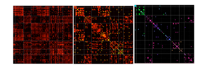
[2] Lee, H., et. al. (2011), ISBI2011.

[3] Goel, et. al. (2005), Annals of Applied Prob., vol. 15.

Network Construction



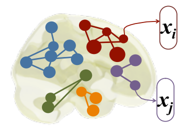
Different adjacency matrices are constructed depending on the thresholds.



Which one do you prefer?

Distance = 1 – positive partial correlation

Sparse Partial Correlation based on Penalized Linear Regression



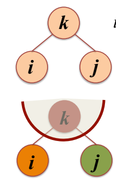
(x_i is centered and normalized.)

- **Correlation** C

  $x_i = \alpha_{ij} x_j + \varepsilon_{ij}$

- **Partial Correlation** C^{-1}

  $x_i = \sum_{i \neq j} \beta_{ij} x_j + \varepsilon_i$



i, j and k are correlated.

i and j are not correlated due to partialing out the effect of k.

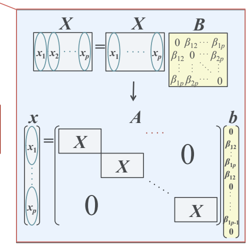
$$x = Ab$$

Dimension of x = Number of Observations ^ Dimension of b = Number of ROIs

Penalize b.

$$\|x - Ab\|^2 + \lambda \|b\|_1$$

least absolute shrinkage and selection operator (LASSO),



- **Coordinate descent**

$$\mathcal{G}_1 = \sum_{i=1}^{p} \| f_i - \sum_{j \neq i} \theta_{ij} \sqrt{\frac{\pi_{jj}}{\pi_{ii}}} f_j \|_2^2 + \lambda \sum_{i,j} |\theta_{ij}| \sqrt{\frac{\pi_{jj}}{\pi_{ii}}}$$

$$\frac{\partial \mathcal{G}_1}{\partial \theta_{ij}} = \begin{cases} A\theta_{ij} - \mathcal{B}(\theta_{ik,k \neq i,j}) + \lambda, & \text{for } \theta_{ij} > 0 \\ A\theta_{ij} - \mathcal{B}(\theta_{ik,k \neq i,j}) - \lambda, & \text{for } \theta_{ij} < 0 \end{cases}$$

$$A = \frac{\pi_{jj}}{n\pi_{ii}} f_j^\top f_j$$

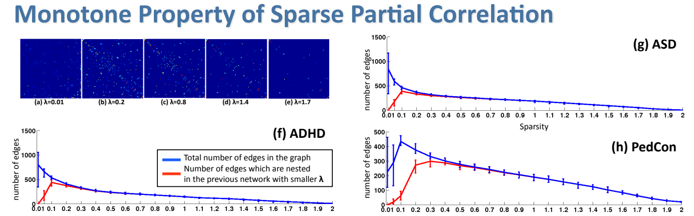
$$\mathcal{B}(\theta_{ik,k \neq i,j}) = \frac{1}{n}\Big(f_i^\top f_j - \sum_{k \neq i,j} \theta_{ik} \frac{\sqrt{\pi_{kk}\pi_{jj}}}{\pi_{ii}} f_k^\top f_j\Big)$$

$$\hat{\theta}_{ij} = \frac{[\mathcal{B}(\theta_{ik,k \neq i,j}), \lambda]_+}{A}$$

$$[a, b]_+ = \begin{cases} a - b & \text{if } a > 0 \text{ and } |a| > b \\ a + b & \text{if } a < 0 \text{ and } |a| > b \\ 0 & \text{if } |a| \le b \end{cases}$$

(Threshold function which takes the value a, if $|a| > b$, and makes zero, otherwise.)

Monotone Property of Sparse Partial Correlation



(a)-(e) show the partial correlation maps for different λ = 0.01, 0.2, 0.8, 1.4 and 1.7. The number of edges in the network changes as λ value changes for ADHD (f), ASD (g) and PedCon (h). The blue plots are the total numbers of edges in the network. The red plots are the numbers of edges which are included in the previous network with smaller λ among all edges in a graph.

Barcodes of Sparse Brain Network