Fourier Spectral Method for Shape Asymmetry Analysis

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Acknowledgments

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Abstracts

Although shape asymmetry has been investigated in many branches of science, there is a lack of unified methodological framework for quantifying local shape asymmetry. Previous literature mainly deals with quantifying a global amount of shape asymmetry. A more interesting question would be to ask if we could spatially localize the source of asymmetry. In brain imaging, this question has been successfully addressed by using the deformable template approach of Grenander and Miller. By registering the original and its mirror reflected 3D magnetic resonance image (MRI), one can establish the correspondence across brain hemispheres and, in turn, able to construct the localize asymmetry index of type (L-R)/(L+R). The additional computational burden of establishing deformation across hemispheres and possible mismatching of sulcal pattern across subjects are two major shortcomings of this widely used approach. In this talk, we present a different framework for shape asymmetry analysis that basically combines the deformable template idea and Brechbuler's 3D Fourier descriptor. Surface shape registration, surface data smoothing and surface parameterizations are all tackled in a unified framework. This is a joint work with Kim Dalton and Richard Davidson of the Waisman Laboratory for Brain Imaging and Behavior. An application of the same technique to longitudinal mandible shape modeling (in collaboration with Houri Vorperian of the Vocal Tract Development Laboratory) on 300 subjects will be also briefly discussed.
Motivation: quantify abnormal brain structural asymmetry across hemispheres in a group of autistic subjects
Previous 3D approach

1. Image registration across subjects via a template

2. Image registration across hemispheres by registering the original MRI and its mirror reflection.

3. Construct asymmetry index at each voxel.

4. Feed the index into a statistical model.
Two population asymmetry analysis framework

Clinical population

Normal controls

template

image registration
Three issues with this well established 3D approach

1. 3D image registration can easily misalign sulcal pattern.

2. Mirror reflection and doing image registration is an additional computational burden.

3. The 3D approach does not work for 2D cortical surface data. New 2D framework is needed.
Comparison of surface registration on 149 subjects

Left central & temporal sulci

Right central & temporal sulci


3D registration  2D registration
Probability of matching in the right central sulcus

3D volume registration

2D surface registration
Literature vs. new framework

- **Surface data smoothing**
  - Diffusion smoothing (Neurolmage, 2003)
  - Heat kernel smoothing (Neurolmage, 2005)

- **Surface parameterization**
  - SPHARM
    - Guido Gerig
    - Martin Styner
    - Li Shen

- **Surface registration**
  - PDE
    - Paul Thompson
    - Michael Miller

- **Multiple comparison correction**
  - Random field theory
    - Keith Worsley
    - Jonathan Taylor

**New unified approach:**
Weighted spherical harmonic representation (TMI, 2007)
Outline of talk

1. Introduction to cortical surface data
2. Weighted Fourier series representation
3. Surface registration
4. Surface asymmetry index
5. Statistical analysis
6. Future research direction
Data: 3T MRI
16 high functioning autistic subjects (15.93±4.71 years)
12 normal controls (17.08±2.78 years)
Right-handed males of compatible age range.
Our method

82,190 triangles
40,962 vertices
20,000 parameters per surface

Polygonal mesh
Mesh resolution 3mm
Cortical surface flattening

- Deformable surface algorithm (McDonalds et al., 2001) is used to segment surfaces and obtain the mapping from a unit sphere to a cortical surface.

- Functional measurement defined on cortical surface will be pulled back onto the unit sphere.
Cortical surface flattening as an inverse process of the deformable surface algorithm.
Example of functional measurement pulled back onto unit sphere

Sum of principal curvatures

Cortical flat map

**Note:** metric distortion might influence the final statistical analysis.
There is a way to address area distortion. Local area element can be obtained by analytically differentiating WFS and computing metric tensors. This measures amount of area distortion associated with cortical flattening. It can be used as a nuisance covariate in a statistical analysis.
Cortical manifold and function measurement defined on the manifold

Anatomical manifold $\mathcal{M} \in \mathbb{R}^d$

Parameter space $\mathcal{N} \in \mathbb{R}^m$

Parameterization $u : \mathcal{N} \rightarrow \mathcal{M}$

Hilbert space $L^2(\mathcal{N})$ with inner product

$$\langle g_1, g_2 \rangle = \int_{\mathcal{N}} g_1(p)g_2(p)\mu(p)$$

Self-adjoint operator $\mathcal{L}$

$$\langle \mathcal{L}g_1, g_2 \rangle = \langle g_1, \mathcal{L}g_2 \rangle$$

Basis function

$$\mathcal{L}\psi_j = \lambda_j \psi_j$$
Weighted Fourier Series (WFS) representation

cortical thickness (function)
+ surface coordinates (surface)

\[ \partial_t g + \mathcal{L} g = 0, \quad g(p, t = 0) = f(p) \]

PDE

solution

Spectral representation

\[ g(p, t) = \sum_{j=0}^{\infty} e^{-\lambda_j t} \langle f, \psi_j \rangle \psi_j(p) \]

Kernel smoothing

\[ = \int_{\mathcal{N}} K_t(p, q) f(q) \, d\mu(q) \]
Spherical harmonic of degree $l$ and order $m$

$$Y_{lm} = \begin{cases} 
  c_{lm} P_l^{|m|} (\cos \theta) \sin(|m|\varphi), & -l \leq m \leq -1, \\
  \frac{c_{lm}}{\sqrt{2}} P_l^0 (\cos \theta), & m = 0, \\
  c_{lm} P_l^{|m|} (\cos \theta) \cos(|m|\varphi), & 1 \leq m \leq l, 
\end{cases}$$
WFS = parameterization + smoothing

Original cortical surface

Coordinate functions

x
y
z

Weighted-SPHARM

45 mm
-45 mm
WFS = multiscale representation

Color scale = x-coordinate
WFS can be applied to functional data like cortical thickness.

Cortical thickness = most widely used cortical structural measure
What is cortical thickness?

• distance between surfaces
• measures amount of gray matter bounded by these two surfaces
WFS of cortical thickness

Cortical thickness

1st row:
\[ \sum_{l=0}^{k} \sum_{m=-l}^{l} e^{-l(l+1)t} \langle f, Y_{lm} \rangle Y_{lm}(\theta, \varphi) \]

2nd row:
\[ \sum_{m=-k}^{k} e^{-l(l+1)t} \langle f, Y_{lm} \rangle Y_{lm}(\theta, \varphi) \]

Images showing different cortical thickness maps with varying $k$ values.
Iterative residual fitting (IRF) algorithm (TMI, 2007)
Can estimate more than 20,000 coefficients per surface
Joint work with Li Shen

Step 1. measurements \( f(p_1), \ldots, f(p_n) \)

Step 2. Set initial degree=0 \( k = 0 \)

Step 3. Solve \( f(p_i) = \sum_{m=-k}^{k} \beta_{km} Y_{km}(p_i) \)  
Project data into a finite subspace

Step 3.5. \( f \leftarrow f - \hat{f} \)  
Once low frequency parts are estimated, we throw them away

Step 4. Set degree \( k \leftarrow k + 1 \)

MATLAB code available at  [http://www.stat.wisc.edu/~mchung/](http://www.stat.wisc.edu/~mchung/)

MR is identical to IRF in principle except the methods for estimating Fourier coefficients are different.

IRF was developed to solve a large linear system (with orthonormality contrain) iteratively.

MP was developed as a way to compactly decompose time frequency signal into a linear combination of basis in a dictionary.
Statistical Model on WFS: Karhunen-Loeve expansion

\[
\sum_{l=0}^{k} \sum_{m=-l}^{l} e^{-l(l+1)t} \langle f, Y_{lm} \rangle Y_{lm}(\theta, \varphi)
\]

Uncorrelated normal
Why WFS?
Reduction of Gibbs phenomenon (ringing artifacts)

Functional data defined on sphere

Top: Fourier series expansion (SPHARM)
Bottom: WFS
Why WFS? Gibbs phenomenon
Anatomical data
Surface registration via WFS

Given two $l$-th degree WFS surfaces $v_{i1}, v_{i2}$, find the displacement $d_i$ that minimizes the discrepancy between two surfaces:

$$v_{i2} - v_{i1} = \arg \min_{d_i \in \mathcal{H}_l} \int_{\mathcal{M}} [v_{i1} + d_i(v_{i1}) - v_{i2}]^2 \, d\mu(p).$$

$\mathcal{H}_l$ : subspace spanned by up to $l$-th degree spherical harmonics

$v_{i1} + d_i(v_{i1})$ : deformation of coordinates $v_{i1}$

Consequence: For fixed $(\theta, \varphi)$,

$$v_{i1}(\theta, \varphi) \text{ corresponds to } v_{i2}(\theta, \varphi).$$
Example of surface registration

subject 1

\[ \alpha = 0 \]

\[ v_{i1} + \alpha d_i(v_{i1}) \]

subject 2

\[ \alpha = 1 \]
Cortical asymmetry analysis
Establishing hemispheric correspondence algebraically

Mirror reflection: It is done algebraically on WFS
Surface registration
Invariance under mirror reflection

\[
\hat{g}(\theta, \varphi) = \sum_{l=0}^{k} \sum_{m=-l}^{l} e^{-l(l+1)\sigma} \langle f, Y_{lm} \rangle Y_{lm}(\theta, \varphi).
\]

\[
\hat{g}(\theta, 2\pi - \varphi) = \sum_{l=0}^{k} \sum_{m=-l}^{l} e^{-l(l+1)\sigma} \langle f, Y_{lm} \rangle Y_{lm}(\theta, \varphi)
- \sum_{l=0}^{k} \sum_{m=0}^{l} e^{-l(l+1)\sigma} \langle f, Y_{lm} \rangle Y_{lm}(\theta, \varphi)
\]
Shape decomposition into symmetric and asymmetric parts

Symmetric part

\[ S(\theta, \varphi) = \frac{1}{2} \left[ \hat{g}(\theta, \varphi) + \hat{g}(\theta, 2\pi - \varphi) \right] = \sum_{l=0}^{k} \sum_{m=-l}^{-1} e^{-l(l+1)\sigma} \langle f, Y_{lm} \rangle Y_{lm}(\theta, \varphi) \]

Asymmetric part

\[ A(\theta, \varphi) = \frac{1}{2} \left[ \hat{g}(\theta, \varphi) - \hat{g}(\theta, 2\pi - \varphi) \right] = \sum_{l=0}^{k} \sum_{m=0}^{l} e^{-l(l+1)\sigma} \langle f, Y_{lm} \rangle Y_{lm}(\theta, \varphi) \]

Normalized asymmetry index

\[ N(\theta, \varphi) = \frac{\hat{g}(\theta, \varphi) - \hat{g}(\theta, 2\pi - \varphi)}{\hat{g}(\theta, \varphi) + \hat{g}(\theta, 2\pi - \varphi)} = \frac{\sum_{l=1}^{k} \sum_{m=-l}^{-1} e^{-1(l+1)\sigma} \langle f, Y_{lm} \rangle Y_{lm}(\theta, \varphi)}{\sum_{l=0}^{k} \sum_{m=0}^{l} e^{-l(l+1)\sigma} \langle f, Y_{lm} \rangle Y_{lm}(\theta, \varphi)} \]

Ratio of negative and positive degree expansions
Asymmetry index

Cortical thickness

Weighted SPHARM

Asymmetry index

Symmetry index

Normalized asymmetry index
Multiple comparisons

\[ H_0 : \theta_1(p) = \theta_2(p) \text{ for all } p \in \partial \Omega \]

v.s.

\[ H_1 : \theta_1(p) > \theta_2(p) \text{ for some } p \in \partial \Omega. \]

The above hull hypothesis is the intersection of collection of hypothesis

\[ H_0 = \bigcap_{p \in \partial \Omega} H_0(p) \]

**Type I error**

\[ \alpha = P(\text{reject at least one } H_0(p)| H_0 \text{ true }) \]

\[ = P \left( \bigcup_{p \in \partial \Omega} \{ T(p) > h \} \right) \]

\[ = 1 - P \left( \bigcap_{p \in \partial \Omega} T(p) \leq h \right) \]

\[ = 1 - P(\sup_{p \in \partial \Omega} T(p) \leq h) \]

\[ = P(\sup_{p \in \partial \Omega} T(p) > h). \]

*random field*
$Z(x)$: Stationary isotropic random field in $x \in \Omega \subset \mathbb{R}^N$

$A_z = \{x : Z(x) > z\}$ excursion set

$\chi(A_z)$: Euler characteristic

$P\left( \max_{x \in \Omega} Z(x) > z \right) \approx \mathbb{E}(\chi(A_z))$ (Adler, 1984)
T random field on manifolds

\[ P\left( \max_{x \in \partial \Omega_{atlas}} T(x) \geq y \right) \approx 2 \rho_0(y) + \| \partial \Omega_{atlas} \| \rho_2(y) \]

Euler characteristic density

\[ \rho_0(y) = \int_y^\infty \frac{\Gamma\left(\frac{n}{2}\right)}{(n-1)\pi^{1/2}\Gamma\left(\frac{n-1}{2}\right)} \left(1 + \frac{y^2}{n-1}\right)^{-n/2} dy, \]

\[ \rho_2(y) = \frac{1}{\text{FWHM}^2} \frac{4 \ln 2}{(2\pi)^{3/2}} \frac{\Gamma\left(\frac{n}{2}\right)}{(n-1)^{1/2}\Gamma\left(\frac{n-1}{2}\right)} y\left(1 + \frac{y^2}{n-1}\right)^{-(n-2)/2} \]

Worsley (1995, NeuroImage)

FWHM of smoothing kernel or residual field
WFS is related to heat kernel smoothing

WFS

\[ g(p, t) = \sum_{j=0}^{\infty} e^{-\lambda_j t} \langle f, \psi_j \rangle \psi_j(p) \]

Heat kernel smoothing

\[ = \int_{\mathcal{N}} K_t(p, q) f(q) \, d\mu(q) \]

Shape of heat kernel

Numerical computation
Statistical parametric map
multiple comparison correction via
the random field theory
Validation of WFS against analytical ground truth

Heat kernel smoothing
(2005, NeuroImage)
Next project? Mandible surface modeling

Automatic hole patching is necessary to construct surface topologically equivalent to sphere.

Approximately 20,000 triangle elements
Nonlinear surface registration via curvature matching
Mandible surface modeling

Quadratic fit of 9 male subjects over time in one particular point on the mandible surface  Plan: do this on 300 subjects
Locally varying growth rate modeling

Growth rate (obtained directly from the regression model) projected on average mandible surface

Plan: incorporate gender and other variables into analysis.