1. **Negative binomial distribution with parameter** \( r \) **and** \( p \). Let \( X \) be the number of coin tossing until \( r \) number of heads are accumulated. Then

\[
p(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}, \ x = r, r+1, \ldots.
\]

The geometric distribution is a special case of a negative binomial distribution with \( r = 1 \).

\[
E_X = \frac{r}{p} \ 	ext{and} \ E_Y = \frac{r}{1-p}.
\]

2. **The Banach match problem.** A mathematician carries two match boxes each containing \( n \) matches. The probability of using each match box is \( \frac{1}{2} \). If he found one of the box is empty, what is the probability that the one box contains \( k \) matches?

**Solution.** Let \( A_k \) be the above event. Compute \( P(A_0), P(A_{n-1}), \ldots \) and you will see the pattern.

\[
P(A_k) = 2 \binom{2n-k}{n} \left( \frac{1}{2} \right)^{2n-k}, \ k = 0, 1, \ldots, n.
\]

3. **Hypergeometric distribution.** A box contains \( N \) balls, of which \( m \) are white and \( N - m \) are black. Let \( X \) be the number of white balls in \( n \) draws without replacement.

\[
p(x) = \binom{m}{x} \binom{N-m}{n-x} \binom{N}{n}, \ x = 0, 1, \ldots, \min(n, m).
\]

4. **Two people toss a fair coin** \( n \) **times each.** Find the probability that they throw equal number of heads. **solution.** Let \( E_i \) be the event the both throws \( i \) heads. Then we are computing

\[
P(\bigcup_{i=0}^{n} E_i) = \sum_{i=0}^{n} P(E_i).
\]

Note \( P(E_i) = \binom{n}{i} \left( \frac{1}{2} \right)^n \binom{m}{i} \left( \frac{1}{2} \right)^m \).

This problem can be also solved using random variables. Let \( X \) be the number of heads for the first person and \( Y \) be the number of heads for the second person. Then \( X, Y \sim \text{i.i.d. Binomial}(n, \frac{1}{2}) \).

\[
P(X = Y) = \sum_{i=0}^{n} P(X = i, Y = i)
\]

\[
= \sum_{i=0}^{n} P(X = i)P(Y = i)
\]

\[
= \sum_{i=0}^{n} \left( \binom{n}{i} \left( \frac{1}{2} \right)^n \binom{m}{i} \left( \frac{1}{2} \right)^m \right)^2
\]

From the hypergeometric distribution,

\[
\binom{2n}{n} = \sum_{i=0}^{n} \binom{n}{i} \binom{n}{i}.
\]

Hence, the probability is \( \binom{2n}{n} \frac{1}{2^{2n}} \).

**NOTE:** The first midterm exam will cover lectures 1-9 (upto Chapter 4). **HW 3** due Oct 20. Solve the following 9 problems. Chapter 4 Problems 42, 44, 48, 60, 64, 75, 76. Theoretical Exercises 31, 32.