Stat312: Midterm Makeup Exam I.

Instructor: Moo K. Chung
mchung@stat.wisc.edu

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Answer all questions clearly and circle your final answer. Your answers should be correct up to the second decimal places. One page note and a calculator are allowed. No textbooks, scrap papers or hand-held computers, PDA are allowed. This exam booklet consists of 3 problems and 6 pages.

Name:__________________________________________
Student ID:_____________________________________

Pledge: On my honor, I have neither given nor received unauthorized aids on this examination.

Signature:_____________________________________

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1. Suppose a random sample $X_1, \ldots, X_n$ are coming from a normal distribution with mean $\mu$ and variance $\sigma^2$. We are interested in estimating $\mu$ and $\sigma^2$.

   (1) Write the likelihood function (5pts).

   (2) By maximizing the likelihood function, find the maximum likelihood estimators for $\mu$ and $\sigma^2$. Derive everything (5pts).
(3) Determine if the estimator for $\sigma^2$ in (2) is unbiased (5pts, no point given if (2) is not solved).

(4) What is the moment estimator of $\sigma^2$? Derive everything (5pts).
2. Suppose we toss 10 identical coins and observed 4 heads. Let $p$ be the probability of getting head when a single coin is tossed.

(1) Write the likelihood function. Properly define all variables you are using (5pts).

(2) By maximizing the likelihood function, find the maximum likelihood estimator for $p$. Derive everything (5pts).
(3) What is the maximum likelihood estimator for the probability of getting tail?
3. The following 10 sample observations on breakdown voltage of a particular circuit under certain condition is given.

\[ 62, 50, 53, 57, 41, 53, 55, 61, 59, 64. \]

(1) What is the sample mean and the sample variance of the breakdown voltage? (5pts).

(2) Construct 98% confidence interval for the mean breakdown voltage. What is the statistical assumptions you are making to construct the confidence interval? You may use the following R output (5pts).

```r
> qnorm(1:10/100)
> [6] -1.554774 -1.475791 -1.405072 -1.340755 -1.281552
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