Stat312: Midterm Exam I.

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Answer all questions clearly and circle your final answer. Your answers should be correct up to the second decimal places.

One page note and a calculator are allowed.

No textbooks, scrap papers or hand-held computers, PDA are allowed.

This exam booklet consists of 3 problems and 6 pages.

Name: __________________________________________

Student ID: ______________________________________

Pledge: On my honor, I have neither given nor received unauthorized aids on this examination.

Signature: _______________________________________

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
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</tr>
<tr>
<td>3</td>
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<tr>
<td>Total</td>
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</tbody>
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1. Suppose a random sample $X_1, \ldots , X_n$ are coming from a normal distribution with mean 0 and variance $\sigma^2$. We are interested in estimating $\sigma^2$.

(1) Write the likelihood function (5pts).

(2) By maximizing the likelihood function, find the maximum likelihood estimator for $\sigma^2$. Derive everything (5pts).
(3) Determine if the estimator in (2) is unbiased (5pts, no point given if (2) is not solved).
(4) By matching moments, estimate $\sigma^2$ (5pts).
2. Consider a random sample $X_1, X_2, X_3$ coming from distribution

$$f(x) = \frac{1}{\theta} e^{-x/\theta}, x > 0.$$  

(1) Write the loglikelihood function (5pts).

(2) By maximizing the loglikelihood function, find the maximum likelihood estimator for $\theta$. Derive everything (5pts).
(3) What is the maximum likelihood estimator of $P(X_1 > 1)$. 
3. 49 students take a midterm exam in STAT 312. If $x_i$ is the grade for the $i$-th student, we have 
\[ \sum_{i=1}^{49} x_i = 3675 \text{ and } \sum_{i=1}^{49} x_i^2 = 275,625. \]

(1) What is the sample variance? (5pts).

(2) Construct 6\% confidence interval for the average grade. You may use the following R output (5pts).

```r
> qnorm(1:10/100)
> [6] -1.554774 -1.475791 -1.405072 -1.340755 -1.281552
```