1. For two events $A$ and $B$ with $P(B) > 0$, the conditional probability of $A$ given that $B$ has occurred is defined as

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$ 

2. The Smiths have two children. At least one of them is a boy. What is the probability that both children are boys?

**Solution.** Let $X$ be the number of boys. Then

$$P(X = 2|X \geq 1) = \frac{P(X = 2, X \geq 1)}{P(X \geq 1)} = \frac{P(X = 2)}{1 - P(X = 0)} = \frac{1/4}{1 - 1/4}.$$ 

3. The law of total probability. For disjoint events $A_1$ and $A_2$ with $S = A_1 \cup A_2$,

$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2).$$

4. Bayes’ theorem.

$$P(A_1|B) = \frac{P(B|A_1)P(A_1)}{P(B|A_1)P(A_1) + P(B|A_2)P(A_2)}.$$ 

5. 1/10 of men and 1/7 of women are color-blind. A person is chosen at random and that person is color-blind. What is the probability that the person is male. Assume males and females to be in equal numbers.

**Solution.** Let $M=\text{male}, F=\text{female}, C=\text{color-blind}$. Then

$$P(M|C) = \frac{P(M \cap C)}{P(C)} = \frac{P(C|M)P(M)}{P(C|M)P(M) + P(C|F)P(F)} = \frac{\frac{1}{10} \cdot \frac{1}{2}}{\frac{1}{10} \cdot \frac{1}{2} + \frac{1}{7} \cdot \frac{1}{2}}.$$ 

6. A box contains $w$ white balls, $b$ black balls or $r$ red balls. A ball is chosen at random and if it is either black or red then it is replaced by a white ball and if it is white then it is replaced by a red ball. Now again draw a ball. What is the probability that the second ball drawn is red when the first ball drawn is red? What is the probability that the second ball drawn is white?

**Solution.** Let $W_i, B_i, R_i$ be the event that $i$-th draw is a white, black and red ball respectively. The sample space is given by

$$S = W_1 \cup B_1 \cup C_1 = W_2 \cup B_2 \cup C_2.$$ 

$$P(R_2|R_1) = \frac{r - 1}{w + b + r}.$$ 

$$P(W_2) = P(W_2|W_1)P(W_1) + P(W_2|B_1)P(B_1) + P(W_2|R_1)P(R_1)$$

$$= \frac{w - 1}{w + b + r} \cdot \frac{w}{w + b + r} + \frac{w + 1}{w + b + r} + \frac{b}{w + b + r}.$$ 

7. Two events $A$ and $B$ are independent if $P(A|B) = P(A)$. Obviously $A$ and $B$ are independent if $P(A \cap B) = P(A)P(B)$.

8. Suppose that $A \subset B$ and $P(A) > 0$ and $P(B) > 0$. Are two events $A$ and $B$ independent?

**Solution.** Since $A \subset B$, $P(A \cap B) = P(B)$. The condition for the independence is $P(A \cap B) = P(A)P(B)$. Hence, if $P(B) = 1$, $A$ and $B$ are independent but if $P(B) < 1$, $A$ and $B$ are not independent.

**Assigned Problems** Exercise 2.60, Exercise 2.72

Read Chapter 3. First MIDTERM EXAM will be on FEB 22. 9:30-10:45AM. There will be 4 problems in the midterm.