

Statistics 311: Introduction to Mathematical Statistics

January 25, 2007

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- Some course materials will be uploaded in the course website soon.
<http://www.stat.wisc.edu/Department/Courses/2006.2.html>
- The issue with the course enrollment will be handled soon.
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Aim to understand:

- Concepts: "unit", "population of units", "statistical population" and "sample".
- Difference between "population of units" and "statistical population".
- Sampling issues
- Random number table

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Basic Concepts:

Unit and Population of Units:

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“What are GPAs of students’ currently enrolled in this class?”

- Population of Units:
- Statistical Population:
- Variables/Characteristics:

Example 2:

“Students currently enrolled in this class are right/left-handed?”

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NOTE: From the **same population of units**, we can have **different statistical populations** depending on variables/characteristics.

Population of Units vs Statistical Population:

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Consider the following example.

Example 3:

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What would be some difficulties to collect singer preferences from all the city residents?

Nearly impossible to question all the residents in a large city like New York!
Must necessarily settle for taking a sample.

Sampling Issue:

Example 3 - Continued:

A host of a radio music show announced that she wanted to know which singer is the favorite among city residents. Listeners were then asked to call in and name their favorite singer.

- Those who listen to the particular radio station are already a biased subgroup with similar listening habits.
- Those who take the time and effort to call are usually those who have strong opinions about their favorite.

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- Those who take the time and effort to call are usually those who feel strongest about their opinions.

Then how to get a sample that is more representative of the population of units?

Random Number Table:

How the table is constructed?

Suppose ten balls numbered $0, 1, \dots, 9$ are placed in an urn and shuffled. One is drawn and the digit recorded. It is then replaced, the balls shuffled, another one drawn and the digit recorded. The digits in Table 7 at the end of the textbook were actually generated by a computer that closely simulates this procedure.

Why the table is called random?

The chance mechanism that generated the random number table ensures that each of the single digits has the same chance of occurrence, that all pairs $00, 01, \dots, 99$ have the same chance of occurrence, and so on. Further, any collection of digits is unrelated to any other digit in the table. Because of these properties, the digits are called random.

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Random Number Table:

How to use the random table?

Example 4:

Eighty specialty pumps were manufactured last week. Use Table below to select a sample of size $n = 5$ to carefully test and recheck for possible defects before they are sent to the purchaser. Select the sample without replacement so that the same pump does not appear twice in the sample.

3938	4197	6511
7478	7565	5581
4854	9157	9158
7849	7520	2547
6689	1946	2574
9377	6951	6519
8085	4948	2228
3117	1568	0237
0434	4586	4150
2929	7089	3109

Random Number Table:

- 1 Number the pumps from 1 to 80 to identify.
- 2 Select a row and column from the table randomly, say, row 1 and column 5.
- 3 Read the digits in columns 5 and 6 until five different numbers are selected.

41, 75, 91, 75, 19, 69, 49