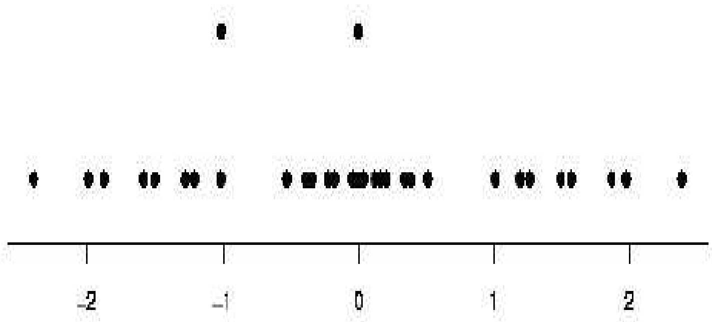
1. Given the five difference D_i = crossed - self, find the permutation distribution of the paired t-statistic.

Crossed	Self	Difference($\frac{1}{8}$) D_i
$23\frac{4}{8}$	$17\frac{3}{8}$	49
12	$20\frac{3}{8}$	-67
21	20	8
22	$18\frac{5}{8}$	16
$19\frac{1}{8}$	$18\frac{3}{8}$	6

```
 [1]  0.12634029 -1.01292199  2.39103417 -0.04203894 -0.21131805  0.00000000
 [7]  0.52120060 -1.26940906 -1.58016033 -1.20116621  1.87006810  1.49579768
[13]  1.98252186 -0.38521957 -0.16871574 -0.34109208  0.34109208  0.16871574
[19]  0.38521957 -1.98252186 -1.49579768 -1.87006810  1.20116621  1.58016033
[25]  1.26940906 -0.52120060  0.00000000  0.21131805  0.04203894 -2.39103417
[31]  1.01292199 -1.01292199
```



2. Prepare 5 cups with milk first and five with tea first. Tell the lady what you have done. If she truly cannot tell difference, find the probability distribution of X = the number of milk first identified. What number(s) would convince you that she could tell the difference?

$$
\begin{aligned}
P(X=5) &= \frac{\binom{5}{5}}{\binom{10}{5}} = \frac{1}{252} = 0.0039 \\
P(X=4) &= \frac{\binom{5}{4}\binom{5}{1}}{\binom{10}{5}} = \frac{25}{252} = 0.0992 \\
P(X=3) &= \frac{\binom{5}{3}\binom{5}{2}}{\binom{10}{5}} = \frac{100}{252} = 0.3968 \\
P(X=2) &= \frac{\binom{5}{2}\binom{5}{3}}{\binom{10}{5}} = \frac{100}{252} = 0.3968
\end{aligned}
$$

$$
\begin{aligned}
P(X=1) &= \frac{\binom{5}{1}\binom{5}{4}}{\binom{10}{5}} = \frac{25}{252} = 0.0992 \\
P(X=0) &= \frac{\binom{5}{0}\binom{5}{5}}{\binom{10}{5}} = \frac{1}{252} = 0.0039
\end{aligned}
$$

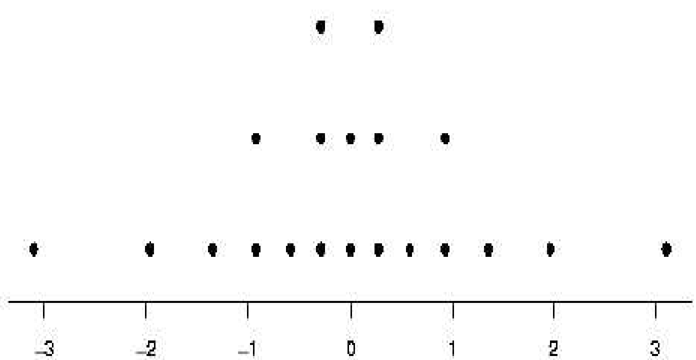
At significant level 0.05, $X = 5$ would be enough.

3. Given the data with $n_1 = n_2 = 3$, find the randomization distribution of

$$t = \frac{\bar{X} - \bar{Y}}{S_{pooled}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad .$$

X	2	4	3
Y	5	7	9

```
 [1] -3.0983867 -1.3522468 -0.5834600  0.0000000 -1.9611614 -0.9258201
 [7] -0.2828427 -0.2828427  0.2828427  0.9258201 -0.9258201 -0.2828427
[13]  0.2828427  0.2828427  0.9258201  1.9611614  0.0000000  0.5834600
[19]  1.3522468  3.0983867
```



4. **Sampling distribution approach**

   (a) Refer to problem 1. Perform a paired t test for equality against a two-sided alternative. That is, refer the observed value of the statistic to the 0.025 quantile of the appropriate t distribution.

$$
\begin{aligned}
H_0 &: \mu_1 = \mu_2 \\
H_1 &: \mu_1 \neq \mu_2
\end{aligned}
$$

$$t = \frac{\bar{d}}{s_d/\sqrt{(n)}} = \frac{2.4}{42.477/\sqrt{5}} = 0.1263 < 2.7764 = t_{4,0.025}$$

Since $|t| < t_{4,0.025}$, we cannot reject H_0.

(b) Obtain 95% confidence interval for $\mu_1 - \mu_2$.

$$\begin{aligned}
(\bar{X} - \bar{Y}) \pm t_{4,0.025} \cdot S_{pooled}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} &= (3-7) \pm (2.7764)(1.5811)\sqrt{\frac{1}{3} + \frac{1}{3}} \\
&= -4 \pm 3.5842 \\
&= [-7.5842, -0.4158]
\end{aligned}$$