

**Practice Final Exam Statistics /M.E. 424**

1 ( 8 points) Three replicates performed where the three factors are labeled A B and C.

A	B	C	Rep. 1	Rep. 2	Rep 3
-1	-1	-1	3	5	4
1	-1	-1	6	4	2
-1	1	-1	11	9	10
1	1	-1	12	15	15
-1	-1	1	10	10	13
1	-1	1	13	11	12
-1	1	1	16	18	17
1	1	1	20	24	22
0	0	0	11	13	12

- (a) 2 points How would you describe this design.
- (b) 2 points Find the estimate of  $\sigma^2$ , the variance of a single observation.
- (c) 4 points Estimate the effects. Use 95% confidence intervals to decide which effects are non-zero.

2 ( 8 points) An experimenter ran an experimental design with two factors Temperature( degrees F) and rate of feed of material into the machine.

	Low	High
<i>Temperature:</i>	150	190
<i>Rate of feed:</i>	5 ft./sec	7 ft/sec

and obtained the estimated response surface

$$\hat{y} = .5x_1 + .8x_2 - 3x_1^2 - 2x_2^2 + 2\sqrt{2}x_1x_2$$

- (a) 2 points Estimate the location of the most extreme response.
- (d) 2 points Express the center in terms of the original temperature and rate of feed.

3 ( 4 points) A  $2^{5-2}$  fractional factorial design is run where the two generators are

$$I = 1234 \quad I = -345$$

(a) 2 points What is the resolution of this design? Explain your answer.

(b) 2 points Give the alias structure for 1 and for 13.

5 ( 9 points) An experiment is run with three factors. The associated MINITAB computer output for the design is

	A	B	C	AB	AC	BC	ABC	AA	BB	CC
1	-1.000	-1.000	-1.000	1	1	1	-1	1	1	1
1	1.000	-1.000	-1.000	-1	-1	1	1	1	1	1
1	-1.000	1.000	-1.000	-1	1	-1	1	1	1	1
1	1.000	1.000	-1.000	1	-1	-1	-1	1	1	1
1	-1.000	-1.000	1.000	1	-1	-1	1	1	1	1
1	1.000	-1.000	1.000	-1	1	-1	-1	1	1	1
1	-1.000	1.000	1.000	-1	-1	1	-1	1	1	1
1	1.000	1.000	1.000	1	1	1	1	1	1	1
1	-1.732	0.000	0.000	0	0	0	0	3	0	0
1	1.732	0.000	0.000	0	0	0	0	3	0	0
1	0.000	-1.732	0.000	0	0	0	0	0	3	0
1	0.000	1.732	0.000	0	0	0	0	0	3	0
1	0.000	0.000	-1.732	0	0	0	0	0	0	3
1	0.000	0.000	1.732	0	0	0	0	0	0	3
1	0.000	0.000	0.000	0	0	0	0	0	0	0
1	0.000	0.000	0.000	0	0	0	0	0	0	0
1	0.000	0.000	0.000	0	0	0	0	0	0	0
1	0.000	0.000	0.000	0	0	0	0	0	0	0
1	0.000	0.000	0.000	0	0	0	0	0	0	0
1	0.000	0.000	0.000	0	0	0	0	0	0	0
1	0.000	0.000	0.000	0	0	0	0	0	0	0
1	0.000	0.000	0.000	0	0	0	0	0	0	0
1	0.000	0.000	0.000	0	0	0	0	0	0	0

The matrix  $(\mathbf{XX})^{-1}$  can be shown to be

$$\begin{bmatrix} 0.167 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.56 & -0.056 & -0.056 \\ 0 & 0.071 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.071 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.071 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.125 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.125 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.125 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.125 & 0 & 0 & 0 \\ -0.056 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.063 & 0.008 & 0.008 \\ -0.056 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.008 & 0.063 & 0.008 \\ -0.056 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.008 & 0.008 & 0.063 \end{bmatrix}$$

The vector of observations is

$$\mathbf{y} = [6, 10, 18, 1, 20, 10, 40, 15, 40, 20, 8, 3, 5, 22, 53, 40, 58, 29, 40, 26]'$$

- (a) 2 points Use the first two columns of the random number table, starting a row 5, to choose the order for the first four runs.
- (b) 2 points Estimate the regression coefficient for the effect of the first factor.(You do not need to find  $\mathbf{X}'\mathbf{y}$ .)
- (c) 2 points Estimate the regression coefficient for the interaction of the second and third factors..
- (d) 2 points A computer program gives

Source	DF	SS
Regression	9	4588.5
Residual Error	10	904.7
Total	19	5493.2

Give a 95 % confidence interval for the coefficient estimated in Part (b).

- (e) 1 point Estimate the variance of an observation using the center points.(This is the pure error estimate)