A Statistical Approach to Dynamic Load Modelling and Identification with High Frequency Measurements

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***Abstract*—As distribution systems become less passive and more complex, accurate dynamic load models are essential to the safe and reliable operation of the network. Dynamic load modelling can be addressed using black-box models, which, while effective in some circumstances, do not provide insight into elements in the network. On the other hand, dynamic load modelling using white-box models can provide insight, but requires extensive knowledge about the number, parameters and type of elements composing the load, which is generally not available. In this work, using white-box modelling, load aggregation, available knowledge of the network, and high frequency measurements, we contribute a statistical methodology to estimate the number, parameters and types of elements composing the load, and also quantify the uncertainty in these estimates. We validate our aggregation technique and estimation framework using simulated data, and test the sensitivity of these results to our underlying assumptions.**

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I. Introduction

Dynamical load modelling is an essential step to efficiently and reliably operate the power network under emerging trends in energy systems [1]. In this paper, we develop a new methodology for dynamic load modelling and estimation, which takes advantage of high-frequency data streams, the matured field of Bayesian statistics, and modern Monte Carlo algorithms. By building on traditional white-box modelling and the hypothesis that the measurements at the bus represent the aggregated response of a large quantity of machines (e.g. [2]), our methodology enables us to (1) discern between different classes of machines, (2) estimate the number of machines in each class, (3) estimate the parameters for each class of machines, and (4) quantify the uncertainty in these estimates. Thus, as we now specify, our methodology provides a more complete picture of the load in comparison to what currently exists in the literature.

In the literature, load modelling is of two types: black-box modelling and white-box modelling (e.g. [3], [4]). The black box load model fits measurement data of past events to a model which need not represent physical characteristics of the load. While such an approach is useful in estimating the behaviour of the load in idealized situations, black-box modelling does not provide insight into the composition of the load, which is useful information in, for example, contingency analyses and planning [5]. On the other hand, white-box modelling uses low-level models of machines to construct the load. For example, in [3], a composite load model is constructed from an induction motor model and a ZIP load model. This white-box model's parameters are fit using bus measurements via likelihood estimation. Unfortunately, the white-box model approach in [3] has two difficulties: (1) the number of components in the load is not considered, and (2) including more components is a non-trivial task owing to the likelihood-based estimation. Another white-box model approach is the WECC model [5]. The WECC model constructs models for each component type, fits these models by direct observations of the component, and combines these components with weights to determine the overall load. As noted in [5], determining these weights and the total number of elements in each component type is difficult, and it appears to be done by trial and error testing.

By building on the successes of these white-box techniques, our modelling and estimation methodology is able to make several improvements. First, it makes use of prior information such as the models and parameters computed in [5]; however, it allows for the parameters to be adjusted based on measurements to better reflect the actual behaviour of the components. Second, it is able to estimate the total number of elements in each class. Third, it is an extremely flexible framework which trivially allows for adding new component types and fitting them using measurement data. Finally, it is able to quantify the uncertainty of the estimates by providing a distribution for the estimated quantities.

II. Methodology

In this section, we describe a computationally feasible load estimation framework, which leverages both measurements and prior load knowledge, in order to identify the composition of the load. We use the following basic notation: $[0,T]$ is a fixed time interval with $T>0$, $v(t)$ is the bus voltage at time $t \in [0,T]$, $I(t)$ is the bus current at time $t \geq 0$, and $\mathcal{A}(\theta, v)$ is the admittance of an element depending on a parameter $\theta(t)$ and voltage $v(t)$. To handle more than one element, let $\mathcal{A}_j$ for $j=1,\ldots,J$ be the admittance of J different types of elements such as an air-conditioner by manufacturer A, an air-conditioner by manufacturer B, lighting, or refrigerators, and let $\theta_{i,j}$ for $i = 1, \ldots, M_j$ be the parameters for the elements of type j. For example, $\mathcal{A}_1(\theta_{1,1}, v)$ and $\mathcal{A}_1(\theta_{2,1}, v)$ are the admittances for two air-conditioning units by a single manufacturer, one with parameter $\theta_{1,1}(t)$ and another with parameter $\theta_{2,1}(t)$. We will often exclude the notation indicating the dependence on time, except when we believe it must be emphasized.

A. Load Estimation Framework

The first step in creating the Estimation Framework is to define a probabilistic model which relates observations of $I(t)$ and $v(t)$ to a first-principles physical model of the load [6, Part II]. The basic physical principle used is Ohm's law:

$$I(t) = v(t) \sum_{j=1}^{J} \sum_{i=1}^{M_j} \mathcal{A}_j(\theta_{i,j}(t), v(t)) \tag{1}$$

We assume that J is approximately known; this assumption is natural as it is known that only a handful of element types account for the bulk of the load, and at most only a handful of manufactures account for these different element types [2], [7]. We also assume that $\mathcal{A}_j$ is a known differentiable, white-box model which accurately captures the admittance behaviour of the element class j up to the desired modelling complexity (e.g. [5]). Finally, since it is known that modelling each element is infeasible, we aggregate the elements belonging to a single class as described in II-B, which reduces (1) to

$$I(t) = v(t) \sum_{j=1}^{J} M_j \mathbb{E}\left[\mathcal{A}_j(\theta_j(t), v(t))\right] \tag{2}$$

where the expectation is evaluated over $\theta_j(t)$, which is now a random variable used to capture the variations in $\theta_{1,j}(t), \ldots, \theta_{M_j,j}(t)$. Therefore, this problem reduces to providing prior distributions for $\tilde{I}(t)$, M_j and θ_j. Since the errors in $\tilde{I}(t)$ are typically device measurements errors of $I(t)$, a natural prior is to model $\tilde{I}(t) \sim Normal(I(t), \sigma^2)$ where σ^2 is the variance of the measurement error, which is typically well known owing to device testing and regulations [8]. On the other hand, a natural prior distribution on M_j, the number of elements of type j, does not exist and will be highly subjective until better guidelines can be established. Finally, the prior distributions for $\theta_j(t)$ can be constructed using distributions centred around tabulated expected values for such priors [7], which we denote $\theta_j^*(t)$. From our experiments, the exact structure of these distributions (e.g. Uniform, truncated Normal, truncated Doubly-Exponential) does not have a noticeable impact, but ensuring that these distributions have a positive support is important.

Once the prior distributions are well established, estimates of the parameters M_j and θ_j and their uncertainty can be computed using (2), periodic observations of $\tilde{I}(t)$ and $v(t)$ on the interval $[0, T]$, and a Markov Chain Monte Carlo algorithm [6, Part III].

B. Load Aggregation

If $\theta_{i,j}(t)$ for $i = 1, \ldots, M_j$ are independent and identically distributed, then, by the strong law of large numbers, the currents in (1) and (2) will be identical as M_j increases to infinity for each $j = 1, \ldots, J$. Thus, given this desirable consistency property, we will assume that $\theta_{i,j}$ for $i = 1, \ldots, M_j$ are independent and identically distributed when constructing our load aggregation technique. Our load aggregation technique makes use of Taylor's theorem and the linearity of expectation to estimate $\mathbb{E}\left[\mathcal{A}(\theta(t), v(t))\right]$. While we specify our aggregation technique for the more difficult case where at least parts of θ has some dynamics given by $\dot{\theta} = g(\theta, v)$ and the remaining component remain fixed, it can be applied more generally. Moreover, under this formulation, there is only uncertainty in the parameter $\theta(0)$, which must be propagated.

Let $\theta^+ = \theta(t+h)$, $\theta = \theta(t)$, $\theta^- = \theta(t-h)$, and $v = v(t+h)$ for $h > 0$ sufficiently small. Let $\mathcal{A}_\theta$ and g_θ denote the derivatives of $\mathcal{A}$ and g with respect to θ. Then, using a Taylor expansion of $\mathcal{A}$ around $\mathbb{E}\left[\theta\right]$,

$$\mathbb{E}\left[\mathcal{A}(\theta^+, v)\right] = \mathcal{A}(\mathbb{E}\left[\theta\right], v) + \left\langle \mathcal{A}_\theta(\mathbb{E}\left[\theta\right], v), \mathbb{E}\left[\theta^+ - \theta\right] \right\rangle$$

up to an additive $\mathcal{O}\left[\left\|\mathbb{E}\left[\theta^+ - \theta\right]\right\|^2\right]$. Using, for example, a forward Euler's step,

$$\mathbb{E}\left[\mathcal{A}(\theta^+, v)\right] = \mathcal{A}(\mathbb{E}\left[\theta\right], v) + h\left\langle \mathcal{A}_\theta(\mathbb{E}\left[\theta\right], v), \mathbb{E}\left[g(\theta, v)\right] \right\rangle \tag{3}$$

up to an additive $\mathcal{O}\left[h^2\right]$. Similarly, for g,

$$\mathbb{E}\left[g(\theta, v)\right] = g(\mathbb{E}\left[\theta^-\right], v) + \left\langle g_\theta(\mathbb{E}\left[\theta^-\right], v), \mathbb{E}\left[\theta - \theta^-\right] \right\rangle \tag{4}$$

up to an additive $\mathcal{O}\left[\left\|\mathbb{E}\left[\theta - \theta^-\right]\right\|^2\right]$. Using, for example, a single sub-iteration of Backwards Euler,

$$\mathbb{E}\left[\theta\right] = \mathbb{E}\left[\theta^-\right] + h\left[I - h g_\theta(\mathbb{E}\left[\theta^-\right], v)\right]^{-1} g(\mathbb{E}\left[\theta^-\right], v) \tag{5}$$

up to $\mathcal{O}\left[h^2\right]$. Hence, using (3)–(5), $\mathbb{E}\left[\mathcal{A}(\theta^+, v)\right]$ can be approximated using previous iterates of $\mathbb{E}\left[\theta\right]$ up to $\mathcal{O}\left[h^2\right]$.

III. Methodology Testing Setup

In order to test the methodology, we must simulate a voltage source and an observed current. Presently, we overview this simulation and how it is used in testing, and address its shortcomings in greater detail subsequently. For the simulation,

- We generate an ideal voltage source (see Figure 1) to which randomly perturbed small-inertia and/or large-inertia (see Table I) motors are connected in parallel.
- Using (6) and (7), we simulate the load response of this population of motors subject to the ideal voltage source, and compute the current using (1).

Then,

- We test the load aggregation methodology by supplying the aggregation scheme the ideal voltage source, number of motors and their expected parameter from Table I. We then compare the load aggregation's current to the current of the population.
- We test the estimation framework by supplying the framework with the voltage source and population's current

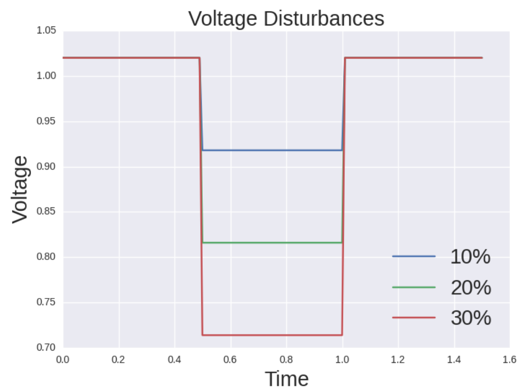

Fig. 1. The voltage profiles used in the load aggregation and estimation framework testing.

at high-frequency in order to simulate measurements generated by a synchrophasor, and then run the Monte Carlo method to infer the total number of elements in the network, the proportion of elements of each motor type (see Table I), and their parameters. We compare these estimates to the true population parameters.

A. Voltage Source

As seen in Figure 1, the ideal voltage source suffers a brief perturbation which exposes the transients in the load. This perturbation is a necessary feature because, otherwise, (1) the load aggregation scheme is trivially able to estimate the total load, and (2) the estimation framework would be unable to distinguish between the small and large motors.

One shortcoming with the ideal voltage source is that better representations of the network voltage can be used, for example, by taking into account network impedances, transformers and generator dynamics. However, the ideal voltage is used here because our present goal is to elucidate the capabilities of the methodology under simple conditions. Another shortcoming is that the ideal voltage source is implicitly assumed to be the terminal voltage for each motor in the population. However, this terminal voltage inaccuracy is readily addressed by adding a voltage noise for each motor.

B. Induction Motors

Induction motors are selected for three reasons. First, induction motors are a primary contributor to the load [2], [9]. Second, because the dynamics of induction motors will be nearly identical, especially for smaller slips, discerning between induction motors with different θ_1^* and θ_2^* is a more difficult task than discerning between two components of different types (e.g. lighting and a motor). Third, discerning between two motor classes has direct applicability in determining the so called "*prone to stall*" motors [9]. Thus, not only does using induction motors gives insight into the limits of applicability of the estimation framework and load aggregation methodology, but it has a small, but relevant application.

The induction motor model used is a first-order single cage induction motor [10], [11] with an equivalent admittance defined by

$$\mathcal{A} = x_m^{-1} + \Big[(r_s + \frac{r_1}{\sigma}) + j(x_s + x_1)\Big]^{-1} \tag{6}$$

$$\dot{\sigma} = \frac{1}{2H}\Big(t_m - \frac{r_1 v^2/\sigma}{(r_s + r_1/\sigma)^2 + (x_s + x_1)^2}\Big) \tag{7}$$

The inertia H, magnetizing reactance x_m, stator impedance $z_s = r_s + jx_s$, rotor impedance $z_1 = r_1 + jx_1$, the nominal torque t_m, and the slip σ, compose the components of θ. The slip dynamics, $\dot{\sigma}$, are the only non-zero component of the parameter dynamics, $g(\theta, v)$. As described in the next subsection, we will simulate populations from the small motors for the aggregation testing and from both types of motors for the estimation framework testing (see Table I).

TABLE I
INITIAL AVERAGE PARAMETERS OF SMALL MOTORS (TYPE A) AND LARGE MOTORS (TYPE B). ALL PARAMETERS ARE P.U IN MACHINE BASE [9].

Type	H	x_m	z_s	z_1	t_m
A	0.28	2.4	$0.050 + j0.087$	$0.053 + j0.082$	1.0
B	1.50	3.8	$0.013 + j0.067$	$0.009 + j0.170$	1.0

C. Population Simulation

Using the induction motor model, a true parameter $\theta^*(0)$ (excluding the slip), and for each of the M motors, the motor's parameter is selected from a uniform distribution between $[0.9\theta^*(0), 1.1\theta^*(0)]$ (excluding the slip). Once the non-slip parameters are selected for each motor in the population, given the initial voltage $v(0)$, the machine is initialized and a slip is computed for the steady-state. Then, each motor's slip is propagated over the time interval $[0, 1.5]$ using Backwards Euler with a time step of $h = 0.01$ sec. After computing the admittance of each motor, the current drawn is computed over the time interval. When two types of motors are used, represented by $\theta_1^*(0)$ and $\theta_2^*(0)$, we simply simulate two populations of size M_1 and M_2 respectively and add the current drawn by both populations to determine the observed current. The parameters of the small and large motors are recorded in Table I.

IV. EXPERIMENTS, RESULTS & DISCUSSION

A. Load Aggregation Experiment

As evident in the load aggregation framework, load aggregation will be the most difficult for the largest voltage disturbance. Thus, we compare the aggregate model's current to the population's current for a disturbance where the voltage decreases 30% for $0.5s$ (see Figure 1). The aggregate load is computed once with the parameter of a small motor (see Table I), denoted θ^*, with a step size of $h = 0.01s$. Then, the estimated current drawn by the aggregate load model is compared to thirty independent populations of size

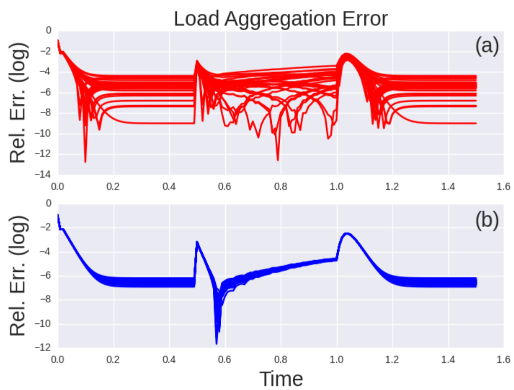

Fig. 2. The relative error for current drawn by populations of size 100 (a) and 50,000 (b) and the current drawn by the load aggregation scheme. As the population size increases, the load aggregate's current is a better estimate of the population's current draw because of the law of large numbers and the concentration of measure phenomenon.

$100, 500, 1e3, 5e3, 1e4, 5e4$ initialized from the parameter θ^*. The relative error between the aggregate model's current and the population current for the populations of size 100 and $50,000$ are displayed in Figure 2.

In general, the aggregate model is able to capture the dynamics of the population rather well, and it improves as the size of the population increases (see Figure 2). The aggregate model's ability to do so depends on two phenomena: the law of large numbers, and concentration of measure [12]. As noted in II-A, the current in (1) should converge to current in (2), which partially explains the aggregate model's ability to better estimate the population's behaviour as the population size increases. However, the tight clustering around the aggregate model is not explained by the strong law of large numbers, as we would expect the variance to grow as $\mathcal{O}[M_j]$ for $j = 1, \ldots, J$. The tight clustering around the aggregate model is a consequence of the boundedness of the population element's parameters, which suggests that a concentration of measure phenomenon is in effect.

B. Load Identification Experiment

We test the estimation framework for a population of 70 small and 30 large motors (see Table I). Since we only have two types of elements, we can reformulate the problem such that instead of M_j we have M as the total number of motors and p as the proportion of Type A motors. The prior distributions for M and p will be $Unif(50, 120)$ and $Unif(0, 1)$ respectively, which are rather uninformative choices and demonstrate the methodology's robustness to the structural and parameter choices for the prior distributions. For the estimation we employ a Sequential Monte Carlo (S.M.C) technique of [13], [14] with 1000 to 1500 particles.

1) Identification of load from dynamic transients: As a simple proof of concept, we simulate 70 motors Type A and 30 motors Type A with higher inertia ($H = 1.4$ p.u). The intuition between the estimation technique is that higher proportions of motors with low inertia will reveal a faster transient than a lower proportion, where the transient is dominated by the higher inertia motors Figure 3. In this case we rely only on the dynamic characteristic of the motors to discriminate between populations. In general, as we increase the degree of disturbance, the framework's ability to discern the total number of elements and the proportion of parameters improves in the sense that the the posterior estimates concentrate more tightly around the true values; therefore, we will discuss the hardest case – the 10% voltage drop case. In Figure 4, we plot the joint posterior (i.e. after using the data) probability of the M and p jointly. Notice, as M approaches the true value of 100, the posterior probability increases dramatically regardless of p. Moreover, as p approaches the true value of 0.7, the posterior probability achieves its maximum. Thus, even in the case of a small drop in potential, we are able to recover the total number of elements and the proportion of small motors rather accurately. However, we expect that as the degree of the disturbance decreases, the posterior will become less and less informative until it coincides with the prior distribution in the limit of a stable potential.

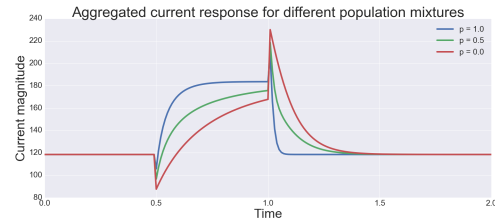

Fig. 3. Current transient for different mixtures of small and large inertia motors. Populations with higher proportion of small motors (blue) will show a faster transient.

2) Effect of prior parameter information: The last experiment reveals the importance of including any information that we have about the load into out framework. Four scenarios are simulated:

1) Estimation of mixture of Type A motors with different inertia, no disturbance.
2) Estimation of mixture of Type A motors with different inertia, 30% voltage disturbance.
3) Estimation of mixture of Type A and Type B motors, no disturbance.
4) Estimation of mixture of Type A and Type B motors, 30% voltage disturbance.

In the case of motors where the only difference is in the dynamic components, we can see that an estimation without a disturbance produces a flat likelihood for all the proportion values, indicating that no specific value is more likely than another. Only a disturbance will expose the dynamic characteristic and give us information to characterize the load. However, for the case of motors of different parameters (Type A and B) using prior parameter information will already reveal some structure of the load.

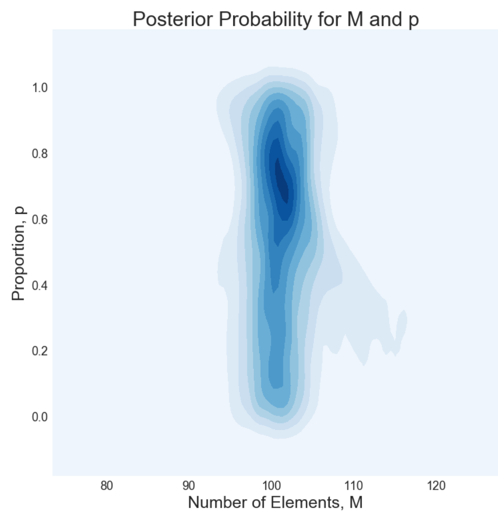

Fig. 4. A smoothed density plot for the joint posterior probability of the total number of elements, M, and the proportion of small motors, p. Darker areas represent higher likelihood, which correspond to the true parameters of $p = 0.7$ and $M = 100$. Note that the smoothing causes the density to take values beyond the acceptable $[0, 1]$ range for the proportion p, not because of the Monte Carlo computation.

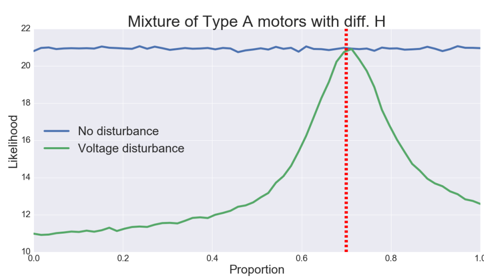

Fig. 5. In the case of motors where the only different is in the dynamic components, only a disturbance will expose the dynamic characteristic and give us information to characterize the load. Notice how the likelihood only "peaks" at the right proportion (in red) after a disturbance.

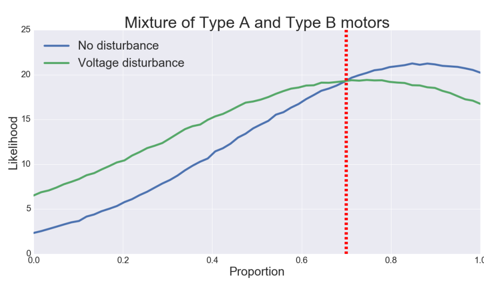

Fig. 6. Prior information allows us to have some estimate of the proportion. But it is not good due to model and measurement errors. The addition of measurements from a transient event makes our estimate better.

V. Conclusion

In this work, we proposed a Bayesian framework for overcoming the challenges of white-box modelling, specifically by incorporating prior knowledge about components in the network to (1) estimate the parameters of component models, (2) estimate the mixture of components in the load, and (3) estimate the total number of components in the load. Further, we introduced and incorporated an aggregation method which ensures that the Bayesian estimation framework is computationally feasible. We validated both techniques using simulated data, and demonstrated that the methodology allows for distinguishing between two rather similar classes of induction motors. In the future, we will extend this methodology in several ways: (1) we will carefully study the impact of smaller potential disturbances on the framework's ability to estimate the unknown quantities, (2) we will implement the framework using real PMU measurements, (3) we will include more elements (e.g. lighting) into the framework, and (4) we will include external influences and data, such as temperature and radiance, into the low level physical models to create a more practical technology.

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