Brief review of multiple imputation

Here we briefly review the underlying Bayesian motivation of multiple imputation, which provide a general rationale of the procedure. Please refer to [1] for rigorous derivation and discussion. Ignoring technical details, denote the complete data by \( Y = (Y_{obs}, Y_{mis}) \), where \( Y_{obs} \) is observed and \( Y_{mis} \) is missing. Let \( Q \) be our interested statistic. One generates \( m \) complete data through \( m \) imputations and obtains the estimates of \( Q \), namely \( \hat{Q}_1, \cdots, \hat{Q}_m \), and the corresponding variance estimation, namely \( U_1, \cdots, U_m \), on the \( m \) individual imputed data.

The basic Bayesian result is

\[
P(Q|Y_{obs}) = \int P(Q|Y_{obs}, Y_{mis})P(Y_{mis}|Y_{obs})dY_{mis},
\]

meaning that the posterior distribution of \( Q \) is the average over the repeated imputations drawn from the posterior predictive distribution of missing data given the observed, i.e. \( P(Y_{mis}|Y_{obs}) \). Therefore, the final estimate of \( Q \) is

\[
E(Q|Y_{obs}) = E[E(Q|Y_{obs}, Y_{mis})|Y_{obs}],
\]

with

\[
\bar{Q} = \frac{1}{m} \sum_{i=1}^{m} \hat{Q}_i
\]

as an unbiased estimate. The final variance of \( Q \) is

\[
V(Q|Y_{obs}) = E[V(Q|Y_{obs}, Y_{mis})|Y_{obs}] + V[E(Q|Y_{obs}, Y_{mis})|Y_{obs}],
\]

which can be estimated by \( \bar{U} + B \) where

\[
\bar{U} = \frac{1}{m} \sum_{i=1}^{m} \hat{U}_i, \quad B = \frac{1}{m-1} \sum_{i=1}^{m} (\hat{Q}_i - \bar{Q})^2.
\]

The above variance estimation is an approximation to the case with infinite number of imputations. To reflect the extra variability of \( \bar{Q} \) for modest \( m \) based on a finite \( m \), we have the following motivation

\[
V(\bar{Q}) = E[V(\bar{Q}|Y_{obs})] + V[E(\bar{Q}|Y_{obs})].
\]

For \( E[V(\bar{Q}|Y_{obs})] \), \( B/m \) is an unbiased estimate. We could estimate \( V[E(\bar{Q}|Y_{obs})] \) by \( \bar{U} + B \). Therefore, the total variance estimation is given by \( T = \bar{U} + (1+\frac{1}{m})B \).

Assume that with complete data, inferences for \( Q \) would be based on the statement that \( (Q - \bar{Q}) \sim N(0, U) \) where \( \bar{Q} \) is a statistic estimating \( Q \), \( U \) is a statistic providing the variance of \( Q - \bar{Q} \). From this, one is able to construct \( t \) reference distribution for \( (Q - \bar{Q})T^{-1/2} \) with small \( m \) and approximately normal
distribution with large $m$. In practice, it is common to choose $Q$ to be some one-to-one function of the quantity of interest that makes the normal approximation reasonable. For LEF, the asymptotic normality is indeed validated for backward imputation procedure when no covariate exists which reduces to the asymptotic result of Kaplan-Meier estimator, and when the base model is specified to be kernel regression which reduces to the asymptotic result established in [2]. For our application to BDES, the empirical results in Figure 9 provide evidence for the normal assumption. Moreover, experience has repeatedly shown that multiple imputation tends to be quite forgiving of departures from the imputation model[3].

**Simulation**

In this section, we present results from three simulation studies that were run to evaluate the performance of the backward multiple imputation method. There were three settings to generate the data. The first corresponds to an additive model with $e(t|x) = t + \exp(-t) + \beta_1 x_1 + \beta_2 x_2$. The second is a proportional model with $e(t|x) = t + \exp(-t + \beta_1 x_1 + \beta_2 x_2)$. The third is a hybrid model with $e(t|x) = t + \beta_1 x_1 + \beta_2 x_2 + \exp(-t + \beta_3 x_3 + \beta_4 x_4)$. We generated the censoring variable $C$ from exponential distribution with rate parameter that resulted in 30% right censoring rate. The sample size was $n = 300$ for the first two settings and was 400 for the third one. The covariates are mutually independent. $x_1$ and $x_3$ were drawn from Bernoulli(1/2) and $x_2$ and $x_4$ were drawn from Uniform(0, 2) with $\beta_1 = 1, \beta_2 = 0.5, \beta_3 = -1$ and $\beta_4 = -0.5$. The base model was linear regression for simulation 1 and was SS-ANOVA for the remaining two settings. We assumed the errors followed a normal distribution for all three cases. This means our posterior predictive distribution from which we drew samples to impute the censored cases is also a normal distribution. The multiple imputation number was set to be 20 and we repeated for 300 times for all three settings. The life expectancy functions were estimated for 0.1, 0.25, 0.5, 0.75 and 0.9 quantiles of the censored survival time $Y$ given all covariates values fixed at 1. We examined the estimated bias, standard deviation and empirical coverage of 95% confidence intervals. (We used normal approximation since $v_m$’s calculated according to algorithm 3 are large.) Table 3 summarized the results and it showed that the backward multiple imputation framework with tailored base model performed pretty well for estimating $e(t|x)$. Moreover, it also gave the desired coverages for the true $e(t|x)$ using the 95% normal confidence interval with the variance estimation based on the multiple imputation idea.

**Proof of Theorem 1**

**Proof 1** It is easy to see that both $\hat{e}_B(t)$ and $\hat{e}_{KM}(t)$ are step functions. So we only need to prove that both functions jump at the same $t$ with the same value. From the explicit expression of $\hat{e}_{KM}(t)$, we know that $\hat{e}_{KM}(t)$ is left continuous
Table 1: Summary of results for estimated life expectancy function using backward imputation method with three different settings.

<table>
<thead>
<tr>
<th>Target point</th>
<th>$Q_{0.1}$</th>
<th>$Q_{0.25}$</th>
<th>$Q_{0.5}$</th>
<th>$Q_{0.75}$</th>
<th>$Q_{0.9}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Additive</td>
<td>-0.0095</td>
<td>-0.0160</td>
<td>-0.0167</td>
<td>-0.0257</td>
<td>-0.0423</td>
</tr>
<tr>
<td></td>
<td>0.1656</td>
<td>0.1770</td>
<td>0.2194</td>
<td>0.3346</td>
<td>0.6199</td>
</tr>
<tr>
<td></td>
<td>0.946</td>
<td>0.948</td>
<td>0.948</td>
<td>0.950</td>
<td>0.950</td>
</tr>
<tr>
<td>Proportional</td>
<td>0.0061</td>
<td>0.0030</td>
<td>-0.0047</td>
<td>-0.0181</td>
<td>-0.0235</td>
</tr>
<tr>
<td></td>
<td>0.0346</td>
<td>0.0369</td>
<td>0.0430</td>
<td>0.0566</td>
<td>0.0813</td>
</tr>
<tr>
<td></td>
<td>0.973</td>
<td>0.963</td>
<td>0.983</td>
<td>0.970</td>
<td>0.987</td>
</tr>
<tr>
<td>Hybrid</td>
<td>0.0207</td>
<td>0.0122</td>
<td>0.0047</td>
<td>-0.0225</td>
<td>-0.0578</td>
</tr>
<tr>
<td></td>
<td>0.1983</td>
<td>0.2057</td>
<td>0.2489</td>
<td>0.3490</td>
<td>0.5504</td>
</tr>
<tr>
<td></td>
<td>0.963</td>
<td>0.963</td>
<td>0.943</td>
<td>0.947</td>
<td>0.940</td>
</tr>
</tbody>
</table>

and is discrete at $t_{(1)}, \cdots, t_{(K)}$. For $\hat{e}_B(t)$ and a particular value $t^*$:

1. If $t^* \notin \{y_1, \cdots, y_n\}$, then $I_{\{y_i > t^*\}}$ is continuous in the neighborhood of $t^*$ for all $i$ and hence $\hat{e}_B(t)$ is continuous around $t^*$.

2. If $t^* \in \{c_{(1)}, \cdots, c_{(J)}\} \backslash \{t_{(1)}, \cdots, t_{(K)}\}$ and denote the number of censored data points at $t^*$ by $n^*$, then $I_{\{y_i > t^*\}} = I_{\{y_i > t^* + \}}$ for all $i$. Hence,

$$\hat{e}_B(t^*) = \hat{e}_B(t^*+) .$$

In addition, since $t^* \in \{c_{(1)}, \cdots, c_{(J)}\} \backslash \{t_{(1)}, \cdots, t_{(K)}\}$, we have

$$\hat{e}_B(t^*) = \frac{\sum_{i=1}^{n} \tilde{y}_i I_{\{y_i > t^*\}}}{\sum_{i=1}^{n} I_{\{y_i > t^*\}}} .$$

and

$$\hat{e}_B(t^*-) = \frac{\sum_{i=1}^{n} \tilde{y}_i I_{\{y_i > t^* - \}}}{\sum_{i=1}^{n} I_{\{y_i > t^* - \}}} = \hat{e}_B(t^*) n^* + \sum_{i=1}^{n} \tilde{y}_i I_{\{y_i > t^*\}} = \frac{\hat{e}_B(t^*) n^* + \hat{e}_B(t^*) \sum_{i=1}^{n} I_{\{y_i > t^*\}}}{n^* + \sum_{i=1}^{n} I_{\{y_i > t^*\}}} .$$

Hence, $\hat{e}_B(t)$ is continuous at times when only censoring occurs.

3. If $t^* \in \{t_{(1)}, \cdots, t_{(K)}\}$ and suppose $t^* = t_{(k)}$ for some $k = 1, \cdots, K$. If
\[ k = K, \text{ it is obvious that } \hat{e}_B(t_{(k)}) = \hat{e}_{KM}(t_{(k)}) = 0. \text{ For } k = 1, \cdots, K - 1, \]
\[
\hat{e}_B(t_{(k)}) = \frac{\sum_{i=1}^{n} \hat{y}_i I_{\{y_i > t_{(k)}\}}}{\sum_{i=1}^{n} I_{\{y_i > t_{(k)}\}}}
= \frac{\sum_{i=1}^{n} \hat{y}_i I_{\{y_i > t_{(k)}\}} + \sum_{i=1}^{n} \hat{y}_i I_{\{t_{(k)} < y_i < t_{(k+1)}\}}}{\sum_{i=1}^{n} I_{\{y_i > t_{(k)}\}}},
\]

Note that within \((t_{(k)}, t_{(k+1)})\), one can only have censored observations. By the backward imputation procedure and analysis on censored times above, we know that

\[
\hat{y}_i = \hat{e}_B(y_i) = \hat{e}_B(t_{(k+1)})
\]

for \(t_{(k)} < y_i < t_{(k+1)}\). Therefore,

\[
\hat{e}_B(t_{(k)}) = \frac{\hat{e}_B(t_{(k+1)}) \sum_{i=1}^{n} I_{\{y_i \geq t_{(k+1)}\}} + \hat{e}_B(t_{(k+1)}) \sum_{i=1}^{n} I_{\{t_{(k)} < y_i < t_{(k+1)}\}}}{\sum_{i=1}^{n} I_{\{y_i > t_{(k)}\}}}
= \hat{e}_B(t_{(k+1)}) - .
\]

The above result shows that \(\hat{e}_B(t)\) jumps only at \(\{t_{(1)}, \cdots, t_{(K)}\}\) and is left continuous.

Suppose there exists an \(l\) such that \(c_{(l)} = t_{(k)}\), i.e. there are both events and censoring happened at \(t = t_{(k)}\) (if \(t_{(k)}\) is a pure event time point, the following analysis still applies by deleting all the terms related with \(c_{(l)}\), then

\[
\hat{e}_B(t_{(l)}) = \frac{\sum_{i=1}^{n} \hat{y}_i I_{\{y_i \geq t_{(k)}\}}}{\sum_{i=1}^{n} I_{\{y_i \geq t_{(k)}\}}}
= \frac{\sum_{i=1}^{n} \hat{y}_i I_{\{y_i > t_{(k)}\}} + n(t_{(k)}) t_{(k)} + n(c_{(l)}) \hat{e}_B(c_{(l)})}{\sum_{i=1}^{n} I_{\{y_i \geq t_{(k)}\}}}
= \frac{\hat{e}_B(t_{(k)}) \sum_{i=1}^{n} I_{\{y_i > t_{(k)}\}} + n(t_{(k)}) t_{(k)} + n(c_{(l)}) \hat{e}_B(t_{(k)})}{\sum_{i=1}^{n} I_{\{y_i > t_{(k)}\}} + n(t_{(k)}) + n(c_{(l)})} . \tag{1}
\]

Now, let’s look at \(\hat{e}_{KM}(t)\). By the explicit formula, we know that for \(k = 1, \cdots, K - 1, \)

\[
\hat{e}_{KM}(t_{(l)}) = t_{(l)} - t_{(l-1)} + \frac{1}{S_{KM}(t_{(l-1)})} \sum_{l=1}^{K} (t_{(l)} - t_{(l-1)}) \hat{S}_{KM}(t_{(l-1)})
\]

\[
\hat{e}_{KM}(t_{(k)}) = t_{(k)} + \frac{1}{S_{KM}(t_{(k)})} \sum_{l=k+1}^{K} (t_{(l)} - t_{(l-1)}) \hat{S}_{KM}(t_{(l-1)}),
\]

Thus, it is easy to see that

\[
\hat{e}_{KM}(t_{(k)}) = \hat{e}_{KM}(t_{(k)}) \hat{S}_{KM}(t_{(k)}) S_{KM}(t_{(k-1)}) + t_{(k)} \left[ 1 - \frac{\hat{S}_{KM}(t_{(k)})}{\hat{S}_{KM}(t_{(k-1)})} \right] . \tag{2}
\]
By the definition of Kaplan-Meier estimator, we have

\[
\frac{\hat{S}_{KM}(t_{(k)})}{\hat{S}_{KM}(t_{(k-1)})} = \frac{\prod_{t_{(i)} \leq t_{(k)}} \left[ 1 - \frac{n(t_{(i)})}{\sum_{i=1}^{n} I_{(y_{i} \geq t_{(i)})}} \right]}{\prod_{t_{(i)} \leq t_{(k-1)}} \left[ 1 - \frac{n(t_{(i)})}{\sum_{i=1}^{n} I_{(y_{i} \geq t_{(i)})}} \right]}
\]

\[= 1 - \frac{n(t_{(k)})}{\sum_{i=1}^{n} I_{(y_{i} \geq t_{(k)})}} = \frac{\sum_{i=1}^{n} I_{(y_{i} > t_{(k)})} + n(c_{(i)})}{\sum_{i=1}^{n} I_{(y_{i} \geq t_{(k)})}} \]  \hspace{1cm} (3)

Putting (2) and (3) together yields the following results:

\[
\hat{e}_{KM}(t_{(k)-}) = \frac{\hat{e}_{KM}(t_{(k)}) \sum_{i=1}^{M} n_{i} I_{(y_{(i)} > t_{(k)})} + n(t_{(k)}) + n(c_{(i)}) \hat{e}_{KM}(t_{(k)})}{\sum_{i=1}^{M} n_{i} I_{(y_{(i)} > t_{(j)})} + n(t_{(k)}) + n(c_{(i)})}.
\]

(4)

By the fact that \( \hat{e}_{B}(t_{(K)}) = \hat{e}_{KM}(t_{(K)}) \) together with (1) and (4), we know that \( \hat{e}_{B}(t_{(k)}) = \hat{e}_{KM}(t_{(k)}) \) for \( k = 1, \ldots, K - 1 \). With the fact that both functions are left continuous step functions with same jump locations, we conclude that \( \hat{e}_{B}(t) = \hat{e}_{KM}(t) \) for all \( t \geq 0 \).

Proof of Theorem 2

Before we jump into the proof, we present the nonparametric estimation \( \hat{e}_{P}(t|x) \) defined in [2]. Let \( K : \mathbb{R}^{p} \rightarrow \mathbb{R} \) be the \( p \)-dimensional kernel function and \( h_{n} \) denotes the bandwidth and

\[
W_{n}(x|h_{n}) = \frac{K(\frac{x-x_{i}}{h_{n}})}{\sum_{j=1}^{n} K(\frac{x-x_{j}}{h_{n}})}
\]

which is the weight of the \( i \)th observation according to the closeness to target point \( x \). Then the generalized Kaplan-Meier estimator[4, 5] takes the form of

\[
\hat{S}_{P}(t|x) = I_{\{t \leq Y_{(n)}\}} \prod_{\{i:Y_{(i)} \leq t\}} \left\{ \frac{\sum_{j=i+1}^{n} W_{n}(j|x|h_{n})}{\sum_{j=1}^{n} W_{n}(j|x|h_{n})} \right\} \delta_{(i)}
\]

where \( Y_{(i)} \) denotes the \( i \)th order statistic, and \( \{\delta_{(i)}, W_{n}(i|x|h_{n})\} \) denote the corresponding censoring indicator and weight of the \( i \)th observation.

Let \( t_{(1)} < \cdots < t_{(K)} \) be the distinct ordered values of the observed survival
time. Then \( \hat{e}_P(t|x) \) by inverting \( S_P(t|x,h_n) \) is given by

\[
\hat{e}_P(t|x) = \begin{cases}
    t_{(k)} + \frac{1}{S_P(t_{(k-1)}|x)} \sum_{l=k+1}^{K} (t_{(l)} - t_{(l-1)}) \hat{S}_P(t_{(l-1)}|x), & t_{(k-1)} < t < t_{(k)} \\
    t_{(k)} + \frac{1}{S_P(t_{(k)}|x)} \sum_{l=k+1}^{K} (t_{(l)} - t_{(l-1)}) \hat{S}_P(t_{(l-1)}|x), & t = t_{(k)}, k = 1, \ldots, K - 1 \\
    0, & t \geq t_{(K)}.
\end{cases}
\]

**Proof 2** For a given covariates value \( x \), \( \hat{e}_B(t|x) \) is derived by using Kernel regression with \( K \) and \( h_n \) as the base model in Algorithm 2. This is equivalent to implement a weighted version of Algorithm 1 with \( W_{n_i}(x|h_n) \) as weights for the \( n \) data points. Notice that \( \hat{S}_P(t|x) \) differs from \( \hat{S}_{KM}(t) \) by imposing weights \( W_{n_i}(x|h_n) \) to the \( n \) cases. Therefore, it is straightforward to prove Theorem 2 by using the proof of Theorem 1 by introducing weights \( W_{n_i}(x|h_n) \) instead of the uniform weights.

**Results for cohort of baseline age 50 in BDES**

We examined the fitted LEF for the group with baseline age of 70 in BDES data previously. Let’s further look at another cohort with baseline age of 50, which contains around 80% of censoring compared to a censoring rate of about 20% for the cohort of baseline age at 70. Figure 5 presents the effects of BMI, education and gender for the subgroup of rich and healthy nonsmokers of baseline age 50. In comparison with Figure 1, it is obvious that the confidence intervals from the multiple imputation method are much wider, resulting in the insignificance of gender although the estimated LEF for males and females are still apart given the rest of the covariates. In addition, the percentage of observed death ages with education greater than 20 is about 10% lower than that the average in this cohort, which further explains why the widths of the confidence intervals correspond to education of 20 are much more inflated in Figure 5. Similar behaviors are also observed in Figure 6 which explores the effects of income, smoking and heart disease. The heavy censoring rate among subjects with baseline age of 50 gives rise to the statistical insignificance of these important covariates.

**Additional variables in BDES**

We have discussed the most important variables associated with longevity in BDES. There are a couple of medical measurements weakly correlated with the survival times in additional to the ones already in the model (*). The variables we further took into accounts are listed in Table 4, and the new SS-ANOVA model are shown in (**). Functions \( f_1 \) to \( f_8 \) are cubic splines, and \( f_{12} \) and \( f_{17} \)
Table 2: Additional variables in the SS-ANOVA model

<table>
<thead>
<tr>
<th>variable</th>
<th>units</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>hdl</td>
<td>mg/dL</td>
<td>high-density lipoprotein cholesterol (serum)</td>
</tr>
<tr>
<td>hgb</td>
<td>g/dL</td>
<td>hemoglobin (blood)</td>
</tr>
<tr>
<td>glucose</td>
<td>mg/dL</td>
<td>glucose (serum)</td>
</tr>
<tr>
<td>crp</td>
<td>mg/L</td>
<td>C-reactive protein</td>
</tr>
</tbody>
</table>

use the tensor product construction.

\[
(\text{imputed}) \text{lastage} = \mu + f_1(\text{baseage}) + \beta_{\text{gender}}I_{\text{gender} = F} + \\
\beta_{\text{smoke}}I_{\text{smoke} = \text{no}} + \beta_{\text{inc}}I_{\text{inc} > 20K} + \\
\beta_{\text{diabetes}}I_{\text{diabetes} = \text{no}} + \beta_{\text{cancer}}I_{\text{cancer} = \text{no}} + \\
\beta_{\text{heart}}I_{\text{heart} = \text{no}} + \beta_{\text{kidney}}I_{\text{kidney} = \text{no}} + \\
f_2(\text{edu}) + f_{12}(\text{baseage} : \text{edu}) + f_3(\text{bmi}) + \\
f_4(\text{hdll}) + f_5(\text{hgb}) + I_{\text{gender} = F}f_6(\text{hgb}) + \\
f_7(\text{glucose}) + f_{17}(\text{baseage} : \text{glucose}) + f_8(\text{crp}).
\]

Again, we use the cohorts with baseline age of 70 to demonstrate the results, illustrated in Figure 7-8. In Figure 5, we observed the positive influence of HDL as well as the harm of high glucose on longevity given all the rest of covariates. Hgb has a quadratic effects when time point t is small with larger sample size, and is interactive with gender as captured in Figure 7. The optimal hgb range is from 12 to 14 and 14 to 18 for females and males respectively. High level of C-reactive protein is a sign in response to inflammation and turns out to be decrease survival as expected. The changes in the corresponding estimated life expectancy are small when the values of these 4 variables run from the best to the worst scenarios, demonstrating their weak effects in addition to the important variables appeared in (**).

Examples of Bootstrapped distribution for estimation of the conditional lifetime expectancy function in BDES

Figure 9 randomly picks 8 cases for the bootstrapped distributions of \(\hat{e}_{\text{BOOT}}(t|x)\) and displays the histograms. It turns out the bootstrapped distributions are all alike normal distributions. It means that a normal confidence interval derived from the bootstrapped distribution is satisfactory. Therefore, the normal confidence intervals by the multiple imputation procedure are justified since \(\hat{e}_{\text{BM}}(t|x)\) and \(\hat{\text{std}}\{\hat{e}_{\text{BM}}(t|x)\}\) are close to \(\hat{e}_{\text{BOOT}}(t|x)\) and \(\hat{\text{std}}\{\hat{e}_{\text{BOOT}}(t|x)\}\) as shown in Table 3.
Information about data access and computer codes

The data used in this paper (Beaver Dam Eye Study- Drs. Barbara Klein, MD, MPH, and Ronald Klein, MD, MPH, Co-PIs) can be accessed by executing a Data Use Agreement between the University of Wisconsin Dept. of Ophthalmology and the institution associated with the researcher making the request, only for the purpose of replicating analyses found in this paper. To obtain a Data Use Agreement for the data in this paper, contact:

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R codes which duplicate the application to BDES in the paper can be obtained by contacting Jing Kong at kong@stat.wisc.edu.

References


Figure 5: Lifetime expectancy function estimation by \textit{bmi}, \textit{edu}, and \textit{gender} for the subgroup with \textit{baseage} = 50, \textit{smoke} = no, \textit{income} \geq 20K and no disease. The x-axis is time $t$ from 50 to 74. The y-axis is $\hat{e}(t|X = x)$. The shaded area presents 95\% normal confidence intervals.
Figure 6: Lifetime expectancy function estimation by smoking, heart disease, and income for the group with base age = 50, gender = F, bmi = 28 (median of the population), edu = 12 (median of the population) and no other disease. The x-axis is time $t$ from 50 to 74. The y-axis is $\hat{e}(t|X = x)$. The shaded area presents 95% normal confidence intervals.
Figure 7: HDL and Glucose effects on expected lifetime for $\text{baseage} = 70$, gender = $F$, smoke = no, $\text{edu} = 12$, $\text{bmi} = 28$, income $\geq 20K$, $\text{hgb} = 14(\text{median})$, $\text{crp} = 2(\text{median})$ and no disease with $t = 70, 75, 80, 85$ and $90$.

Figure 8: HGB and C-reactive protein effects on expected lifetime for $\text{baseage} = 70$, gender = $F$, smoke = no, $\text{edu} = 12$, $\text{bmi} = 28$, income $\geq 20K$, $\text{hdl} = 50(\text{median})$, $\text{glucose} = 95(\text{median})$ and no disease with $t = 70, 75, 80, 85$ and $90$. 
Figure 9: Bootstrapped distributions for 8 randomly selected \( \hat{e}_{BOOT}(t|x) \) out of 6400 combinations of \( t \) and \( x \) for baseline age of 70