

MULTISENSOR ATTITUDE ESTIMATION

Fundamental Concepts and Applications

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Historical Note

FIFTY YEARS OF WAHBA'S PROBLEM

I'm pleased to make a few historical and personal remarks regarding what is known as *Wahba's problem*, since I'm the *Wahba* in question. As I write this in July 2015, it is the 50th anniversary of the publication of the so-named problem I submitted to the *Society for Industrial and Applied Mathematics (SIAM) Review* Problems and Solutions Section [1].

How did a graduate student who was working at IBM come to submit this problem? Well, I was a single mom determined to somehow get a PhD and managed to do that by working at an IBM group in Palo Alto, California, on a Lockheed subcontract, and attend Stanford part time. The project was a super-secret Star Wars satellite project. One day my boss came to me with the question of how to estimate a rotation matrix that would bring direction cosines of a set of stars in the coordinate system on board the satellite to best agree with the known direction cosines of the stars. The classified work involving only knowing those things that you had an actual *need to know*, and my job involved signal in noise detection, assuming that someone else was determining satellite attitude. However, whatever others were doing must not have been satisfactory; otherwise, the Lockheed boss would not have passed it on to IBM. It landed on my desk, possibly because I had the strongest mathematical background in a group of mostly programmers and engineers or possibly because he passed it around widely. At the time, I was taking a course in multivariate analysis at Stanford given by Professor Ingram Olkin, and the topic for one lecture was, what he called *Autonne's theorem*, that an arbitrary real square matrix A can be written as a product UP where U is orthogonal and P is symmetric positive semidefinite. I saw that, that was the key to solving the satellite attitude problem—I wrote the answer up in an IBM technical report and did not hear anything else about it at work, but after several months' wait I obtained permission to submit it as a problem with solution to *SIAM Review*. *SIAM* did not publish the solution of the proposer if there were other solutions proposed, and in this case a solution proposed by Farrell and Stuelpnagel [2] was published exactly 1 year later in July of 1966. That was 1 month after I received my PhD at Stanford and took up a postdoc under my advisor Emanuel Parzen at Stanford. In 1967, I took a position as an assistant professor at the University of Wisconsin–Madison, where I am now celebrating my 48th year on the Madison faculty.

A few years after graduation, I ran into one of my former professors at the University of Maryland-College Park, George Weiss, who told me that he had solved the Wahba's problem before 1965 in a different context that included dimensions higher than three. In preparation for this note, I attempted to find him and get the exact reference in order to set the record straight, but the trail ended with some people who had worked for him at the National Institutes of Health (NIH) before he retired. If he is alive today, he would be about 85. By the way, Professor Olkin's former students threw a magnificent 90th birthday party for him in 2014, and he is still going strong. I can heartily recommend [3], which, among other things, gives a reference to Autonne's theorem dated 1913.

In any case, Wahba's problem is an example of Stigler's law of eponymy, which says, "No scientific discovery is named after its original discoverer." After talking with George Weiss in the late 1960s, I was working on ill-posed inverse problems and was unaware of the extensive development of the satellite attitude problem until about 1987 when I was on sabbatical at Yale and someone came into the office looking for the Wahba of *Wahba's problem*. It reminded me of the anecdote when someone asked Hilbert "What is this Hilbert space?" The basic Wahba's problem has other applications, for example, to diffusion tensor brain imaging [4].

The typical scientist is surely pleased to have his or her name attached to something important, Stigler's law notwithstanding. Ironically, I have been in the academic world for 48 years, published about 150 peer-reviewed papers and a book, with several results, which I (immodestly) believe are important firsts, but to my knowledge the only thing with my name on it is, apparently, not something I did first, or even had the result published, which hopefully is related to the converse of Stigler's law. I thank Hassen Fourati for giving me the opportunity to reminisce a bit, and congratulate him for putting this interesting book together.

Grace Wahba

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