Solutions for Homework 2

1. (a) \( P(6) = 1 - P(1) - P(2) - P(3) - P(4) - P(5) = 0.1 \)
   (b) \( P(1) + P(2) + P(3) + P(4) + P(5) = 1.15 > 1. \) Impossible.
   (c) \( P(6) = 1 - P(1) - P(2) - P(3) - P(4) - P(5) = 0.3 \)
   (d) \( P(5) < 0. \) Impossible.

2. (a) \( A = \{3R, 3R, 3G\}, B = \{1G, 2G, 3G, 4G\}, A \text{ and } B = \{3G\} \)
   i. \( P(A) = \frac{3}{8}, P(B) = \frac{1}{2}, P(A \text{ and } B) = \frac{1}{8} \)
      \( P(A)P(B) = (\frac{3}{8})(\frac{1}{2}) = \frac{3}{16} \neq \frac{1}{8}. \) Thus \( A \) and \( B \) are not independent.
   ii. \( P(A \text{ or } B) = P(\frac{3}{8}) + P(\frac{1}{2}) = \frac{3}{16} + \frac{1}{2} = \frac{3}{4} \neq 1. \) Thus they are not mutually exclusive.
   iii. \( P(A \text{ and } B) = \frac{1}{8} \)

(b) \( B = \{1G, 2G, 3G, 3G\}. \)
If \( ? = 3, \) then \( A = \{3G, 3G, 3R, 3R\} \)
If \( ? \neq 3, \) then \( A = \{3G, 3G, 3R\} \)
\( A \text{ and } B = \{3G\} \)
If \( ? = 3, \) \( P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{\frac{1}{8}}{\frac{3}{8}} = \frac{1}{3} \)
If \( ? \neq 3, \) \( P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{\frac{1}{8}}{\frac{3}{8}} = \frac{1}{3} \)

3. (a) | Male | Female | Juvenile |
    |-----|-------|---------|
    | 4   | 8     | 4       |
    | 6   | 7     | 11      |
    | 10  | 15    | 15      |

   i. \( \frac{15}{40} = 0.375 \)
   ii. \( \frac{4}{40} + \frac{1}{40} = \frac{1}{10} = 0.2 \)
   iii. \( \frac{8}{15/40} = \frac{8}{15} = 0.533 \)

(b) \( P(Q) = \frac{16}{40} = 0.4, P(R) = \frac{10}{40} = 0.25, P(Q \text{ and } R) = \frac{1}{10} = 0.1 \)
\( P(Q)P(R) = (0.4)(0.25) = .1. \) Thus \( Q \) and \( R \) are independent.

4. (a) \( P_X(7) = .4, P_X(1) = .6, \mu_X = 7(.4) + 1(.6) = 3.4 \)
   \( \sigma_X^2 = (7 - 3.4)^2(.4) + (1 - 3.4)^2(.6) = 8.64. \)
   (b) \( P_Y(2) = .5, P_Y(4) = .5, \mu_Y = 3, \sigma_Y^2 = 1. \)
   (c) \( P(X = 7, Y = 4) = P_X(7)P_Y(4) = .2, \)
   \( P(X = 7, Y = 2) = P_X(7)P_Y(2) = .2, \)
   \( P(X = 1, Y = 4) = P_X(1)P_Y(4) = .3, \)
   \( P(X = 1, Y = 2) = P_X(1)P_Y(2) = .3 \)
   (d) \( W = X + Y \)
   \[ \begin{array}{cccc}
   3 & 5 & 9 & 11 \\
   .3 & .3 & .2 & .2 \\
   \end{array} \]
   \( \mu_W = 6.4, \sigma_W^2 = 9.64 = \sigma_X^2 + \sigma_Y^2. \)
   (e) \( T = X - Y \)
   \[ \begin{array}{cccc}
   -3 & -1 & 3 & 5 \\
   .3 & .3 & .2 & .2 \\
   \end{array} \]
   \( \mu_W = .4, \sigma_W^2 = 9.64 = \sigma_X^2 + \sigma_Y^2. \)