## EXAM # 1

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35</td>
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<tr>
<td>2</td>
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<td><strong>Total</strong></td>
<td><strong>100</strong></td>
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1. A geologist measures the mass $X$ (grams) of each specimen in a sample of $n = 850$ rocks in a university collection. The resulting data are then grouped into three classes; the corresponding frequency table is shown below.

<table>
<thead>
<tr>
<th>Mass (grams)</th>
<th>Number of Specimens</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0, 40)</td>
<td>170</td>
</tr>
<tr>
<td>[40, 120)</td>
<td>425</td>
</tr>
<tr>
<td>[120, 240)</td>
<td>255</td>
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</table>

(a) Sketch a graph of the density histogram. You may use the attached graph paper if you wish. **Show all work!**
(b) Using this information, what proportion of specimens weigh between 80-200 grams? Show all work! (5 pts)

(c) Calculate the following summary statistics for these grouped data. Show all work! (6 pts ea)

- quartile masses $Q_1$, $Q_2$, $Q_3$
- mean mass
- standard deviation
2. A bag contains a total of \( n = 25 \) marbles: 4 red, 6 green, and 15 blue. In an experiment, two marbles are to be randomly drawn, one at a time, without replacement, and their respective colors recorded.

(a) Draw a tree diagram for this experiment, clearly labeling all relevant events and their corresponding probabilities. Then use it to answer the subsequent questions. Show all work!

(b) Calculate the probability that both marbles are the same color.

(c) Calculate the probability that either the first marble is red, or the second marble is red.

(d) Calculate the probability that the second marble is not red, given that the first marble is red.

(e) Calculate the probability that the second marble is red, given that the first marble is not red.

(f) Given that either the first marble or the second marble is red, calculate the probability that both marbles are red.
3. In a certain population of households, a health survey is conducted to investigate the smoking habits of parents of young children with respiratory diseases. In the study, the following disjoint prior probabilities are obtained:

- *Only the mother smokes* in 20% of these households.
- *Both parents smoke* in 5% of these households.
- *Only the father smokes* in 25% of these households.
- *Neither parent smokes* in the remaining 50% of these households.

(a) Calculate the probabilities of the events “Mother smokes” and “Father smokes.” **Show all work.** (5 pts)

(b) Are the events “Mother smokes” and “Father smokes” statistically independent? **Justify.** (5 pts)

The investigators are particularly interested in studying childhood asthma in this population. Suppose the following conditional probabilities are also obtained:

- Asthma occurs in 15% of households where *only the mother smokes.*
- Asthma occurs in 60% of households where *both parents smoke.*
- Asthma occurs in 12% of households where *only the father smokes.*
- Asthma occurs in 2% of households where *neither parent smokes.*

(c) Calculate the probability that a randomly selected household from the population has a child with asthma. **Show all work!** (5 pts)

(d) Calculate the posterior probabilities of the following events. **Show all work!** (3 pts ea)

- *Only the mother smokes, given* that asthma occurs.

- *Both parents smoke, given* that asthma occurs.

- *Only the father smokes, given* that asthma occurs.

- *Neither parent smokes, given* that asthma occurs.

(e) Compare these posterior probabilities to their corresponding prior probabilities given above, and interpret the findings of the study in context, via a brief, clear explanation. (8 pts)