EXAM # 1

Please show all work!

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Grade</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
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<td>2</td>
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<td>4</td>
<td>30</td>
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<td>Total</td>
<td>100</td>
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1. A herpetologist measures the length $X$ (mm) of each specimen in a random sample of $n = 100$ lizards of a particular species. The resulting size data are then grouped into four classes; the corresponding frequency table is shown below.

<table>
<thead>
<tr>
<th>Length (mm)</th>
<th>Number of Specimens</th>
</tr>
</thead>
<tbody>
<tr>
<td>[50, 70)</td>
<td>15</td>
</tr>
<tr>
<td>[70, 100)</td>
<td>30</td>
</tr>
<tr>
<td>[100, 140)</td>
<td>40</td>
</tr>
<tr>
<td>[140, 200)</td>
<td>15</td>
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(a) Sketch a graph of the density histogram. You may use the attached graph paper if you wish. **Show all work!** (8 pts)
(b) Using this information, what proportion of specimens measure between 110-150 mm? Show all work! (3 pts)

(c) Calculate the following summary statistics for these grouped data. Show all work! (3 pts ea)

- quartile lengths $Q_1, Q_2, Q_3$
- mean length
- standard deviation
2. An urn contains a total of \( n = 125 \) marbles: 93 black and 32 white. In an experiment, two marbles are to be randomly drawn, one at a time, without replacement, and their respective colors recorded.

(a) Draw a **tree diagram** for this experiment, clearly labeling all relevant **outcomes** and their corresponding **probabilities**. Then use it to answer the subsequent questions. **Show all work!**

(b) Calculate the **probability** of the event \( E = \) “at least one marble is black.”

(c) Calculate the **probability** of the event \( F = \) “at least one marble is white.”

(d) Are the events \( E \) and \( F \) **statistically independent**? Formally justify your answer.

(e) Calculate the **probability** that both marbles are the same color.

(f) Calculate the **probability** that both marbles are black, given that both are the same color.

(g) Calculate the **probability** that both marbles are black, given that at least one of them is black.
3. Al, Bob, and Carla are three friends who often attend parties together. It is known that:

(i) Al attends with probability 0.6
(ii) Bob attends with probability 0.5
(iii) Carla attends with probability 0.4
(iv) If Bob attends, then Al attends with probability 0.7
(v) If Carla attends, then Al attends with probability 0.8
(vi) If both Bob and Carla attend, then Al attends with probability 0.9
(vii) Bob and Carla attend independently of one another.

(a) Calculate the probabilities of each of the following events. **Show all work!**  

- Bob and Carla both attend:
- Al, Bob, and Carla all attend:
- Al and Bob both attend:
- Al and Carla both attend:

(b) Draw a Venn diagram relevant to the events “Al attends,” “Bob attends,” and “Carla attends” in the box below, and clearly label all corresponding probabilities. **Show all work!**  

(c) Calculate the probability that exactly one of the three friends attends. Show all work.  

(d) Calculate the probability that exactly two of the three friends attend. Show all work.  

(e) Calculate the probability that Carla attends, if Al does not attend. Show all work.
4. In a certain population of ladybird beetles, the following **prior probabilities** are known: 0.5 have one spot, 0.2 have two spots, and 0.3 have three spots. However, the population consists of two different species: one native, and the other introduced. The native species accounts for 0.2 of the one-spotted beetles, 0.9 of the two-spotted, and 0.4 of the three-spotted.

(a) Fill in the values of the following probability table, including the row and column marginal sums. 
*Show all work!*

<table>
<thead>
<tr>
<th></th>
<th>One-spotted</th>
<th>Two-spotted</th>
<th>Three-spotted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Native</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Introduced</td>
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</table>

(b) Calculate (show work) the **posterior probability** that a randomly selected **native** specimen has…

- one spot  
- two spots  
- three spots

(c) Compare EACH **prior** probability given above with its corresponding **posterior** probability in (b), and clearly **interpret** each comparison in the context of this scenario.

(d) Calculate (show work) the **posterior probability** that a randomly selected **introduced** specimen has…

- one spot  
- two spots  
- three spots