Outline

Assessing assumptions

The t-test assuming equal variances is

- very sensitive to dependence,
- moderately robust against unequal variance if $n_1 \approx n_2$, but much less robust if n_1 and n_2 are quite different (e.g. differ by a ratio of 3 or more).
- robust against nonnormality.

Corrective actions for 2 independent samples:

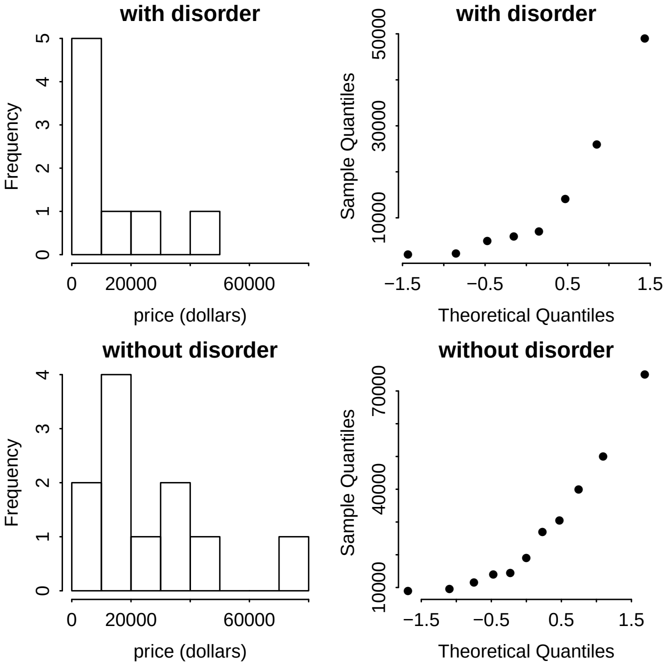
- Fundamental changes if problem with independence (...)
- Welch t-test if σ_1 and σ_2 differ by 3-fold or more or n_1 and n_2 differ by 3-fold or more.
- If non-normal distributions:
  1. try a data transformation,
  2. or switch to a non-parametric test: Mann-Whitney test.

Data transformations

A veterinarian wishes to know if the presence of a certain fetlock disorder in race horses affects their selling price at auction. Data on 8 horses that have the disorder, and 11 that do not (in $)

With Disorder: 5000, 6000, 14100, 49000, 7000, 26000, 2000, 2200

Without Disorder: 27000, 14000, 11500, 19000, 9500, 40000, 75000, 9000, 14500, 50000, 30500


with disorder
Frequency
0
1
2
3
4
5
0
20000
60000
price (dollars)
with disorder
Sample Quantiles
10000
30000
50000
−1.5
−0.5
0.5
1.5
Theoretical Quantiles
without disorder
Frequency
0
1
2
3
4
0
20000
60000
price (dollars)
without disorder
Sample Quantiles
10000
40000
70000
−1.5
−0.5
0.5
1.5
Theoretical Quantiles

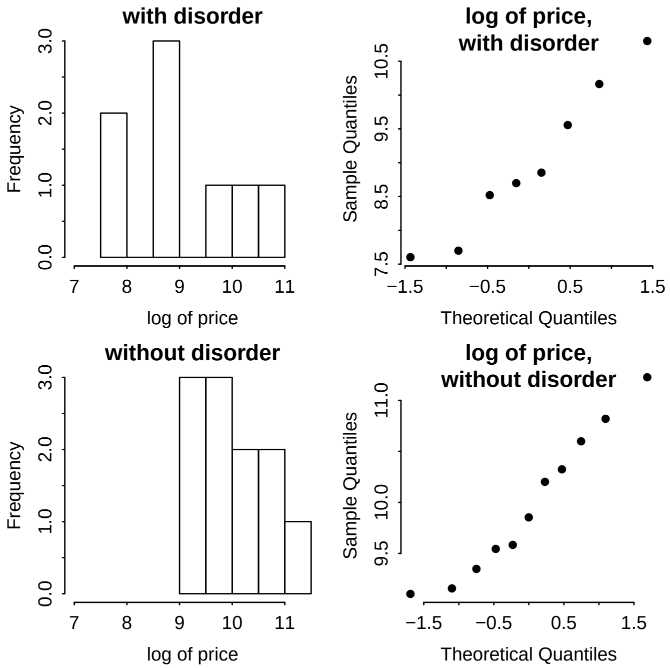
Both samples are skewed right.
Look at the log-values of the prices:

```
> dis
[1]  5000  6000 14100 49000  7000 26000  2000  2200
> log(dis)
[1]  8.52  8.70  9.55 10.80  8.85 10.17  7.60  7.70

> nod
[1] 27000 14000 11500 19000  9500 40000 75000  9000 14500 50000 30500
> log(nod)
[1] 10.20  9.55  9.35  9.85  9.16 10.60 11.23  9.10  9.58 10.82 10.33
```

Could we do the t-test on log-values instead?

If the price tends to go down with the fetlock disorder, then the log(price) also tends to be lower with the disorder than without (and vice versa).


with disorder
Frequency
0.0
1.0
2.0
3.0
7
8
9
10
11
log of price
log of price,
with disorder
Sample Quantiles
7.5
8.5
9.5
10.5
−1.5
−0.5
0.5
1.5
Theoretical Quantiles
without disorder
Frequency
0.0
1.0
2.0
3.0
7
8
9
10
11
log of price
log of price,
without disorder
Sample Quantiles
9.5
10.0
11.0
−1.5
−0.5
0.5
1.5
Theoretical Quantiles

T-test on the log-transformed prices

The distribution of log-prices looks beautifully normal for both samples! Welch t-test on the log-transformed prices:

```
dis = c(5000, 6000, 14100, 49000, 7000, 26000, 2000, 2200)
nod =c(27000, 14000, 11500, 19000, 9500, 40000, 75000, 9000,
       14500, 50000, 30500)

> t.test(log(dis), log(nod))

        Welch Two Sample t-test

data:  log(dis) and log(nod)
t = -2.1955, df = 10.951, p-value = 0.05059
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -1.988749799  0.003048454
sample estimates:
mean of x mean of y
 8.985856  9.978706
```

Conclusion: moderate evidence that the auction prices tend to be lower with the fetlock disorder than without (p=0.051).

Transformations

Log transformation:

helps when the distributions are skewed right,

only when all values are positive

Square-root transformation:

helps when distributions are moderatetly skewed right,

only when all values are ≥ 0 (zeros are okay)

Apply the same transformation (here: take the log) to all values in both samples.

Choose the transformation in order to satisfy assumptions, not based on the resulting p-value.

Confidence intervals on the original scale (not log, not transformed) are more difficult to get.

What if...

The data are too skewed and no transformation can help?

For instance: a transformation might help make one sample look normally distributed but make the other sample look worse.

Third option: use a ‘non parametric’ test, here test that does not assume the normal distribution: the Mann-Whitney test.

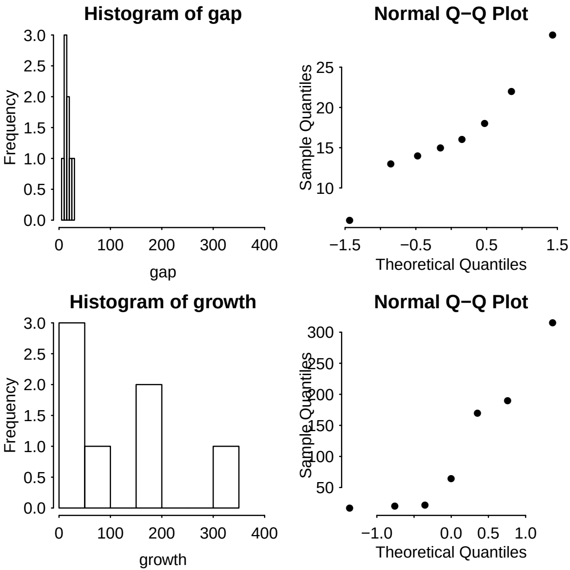
Mann-Whitney test (aka Wilcoxon rank sum test)

Analogous to the Wilcoxon signed-rank test (for paired samples) but here for two independent samples.

- No distribution assumption, but still assume independence.
- Main idea: look at the ranks of the observations

Example: Does soil respiration affect plant growth? Soil cores taken from 2 locations in a forest: under an opening in the forest canopy (“gap”) and at a nearby area under heavy tree growth (“growth”). Measured: amount of carbon dioxide given off by each soil core (mol CO_2/g soil/hr). Data:

Gap	22	29	13	16	15	18	14	6
Growth	17	20	170	315	22	190	64	



Gap data: distribution has normal shape,

Growth data: skewed right.

Welch t-test not recommended, but there is another way!

```
pdf("lec15-01.pdf",width=5,height=5)
gap   =c(22,29,13,16,15,18,14,6)
growth=c(17,20,170,315,22,190,64)
layout(matrix(1:4,2,2))
par(mar=c(3.1,3.1,1.5,.5), mgp=c(1.8,.4,0), tck=-0.01, las=1,bty="n")
hist(gap,    xlim=c(5,400))
hist(growth, xlim=c(5,400), breaks=10)
qqnorm(gap   ,pch=16)
qqnorm(growth,pch=16)
dev.off()
```

Mann-Whitney test

1. H_0: the 2 populations have the same distribution.
   Soil respiration has the same distribution in the 2 locations, with $\mu_1 = \mu_2$ in particular.

   H_A: soil respiration does not have the same distribution in the 2 populations. Test most sensitive to a shift between the 2 distributions, so it's usually assumed that H_A is: 'the 2 distribution have different means'.

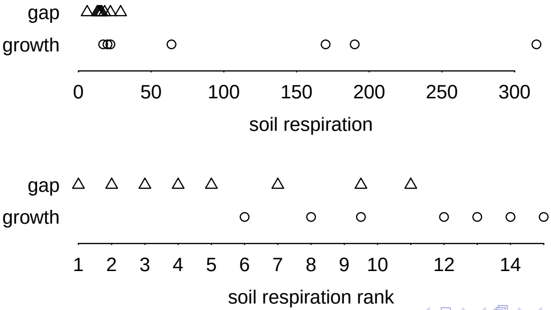
Mann-Whitney test

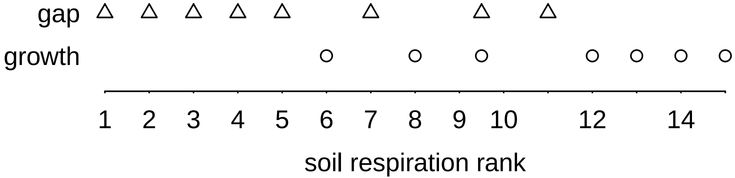
2. Rank the observations, calculate:

   $U_1 =$ # of observations in group 2 that are smaller
   $U_2 =$ # of observations in group 1 that are smaller

   and summarize the data by $U = \max\{U_1, U_2\}$.

   If H_0 is true, then U has a Wilcoxon distribution (does not depend on the common distribution of the data).





U_1 (gap) $= 0 + 0 + 0 + 0 + 0 + 1 + 2.5 + 3 = 6.5$
U_2 (growth) $= 5 + 6 + 6.5 + 8 + 8 + 8 + 8 = 49.5$ so $U = 49.5$.
(for ties: count 0.5)
To double-check: $U_1 + U_2 = n_1 * n_2$ always.
Here yes: $49.5 + 6.5 = 7 * 8 = 56$

If H_0 is true: assignment of ranks to sample is completely random, so expectation: U_1 and U_2 should be similar, i.e. both intermediate, i.e. both about $n_1 * n_2/2$ ($= 28$ here).
$U = \max\{U_1, U_2\}$ expected to be moderate.

More extreme in the direction of H_A: imbalance between U_1 and U_2 (one small, one large), i.e. large U.

Mann-Whitney test

3. We got $U = 49.5$, more extreme $=$ larger, so p-value$=\mathbb{P}\{U \geq 49.5\}$.

   Table E, $n_1 = 8$ and $n_2 = 7$: critical (minimum) U is 46 for rejecting at $\alpha = 0.05$, and 50 at $\alpha = 0.01$

   So here $.01 <$ p-value $< .05$
4. Conclusion: we have moderate evidence that the soil respiration distribution differs between the two locations.

   Soil respiration has a higher mean in the aread under heavy tree growth, than in the area under the opening of the forest canopy.

Note: Table E has no number listed for $n_1 = 3$ and $n_2 = 4$: we can never reject H_0 at $\alpha = 0.05$.

One-sided Mann-Whitney test

H_A: distribution shift with $\mu_1 > \mu_2$ for instance.

First check that the data go in the same direction as H_A, i.e. check that $U_1 > U_2$ if testing H_A: $\mu_1 > \mu_2$.

If not: p-value > 0.50.

If so: p-value is half as much as what it would be for a two-sided test.

wilcox.test() in R

```
> gap
[1] 22 29 13 16 15 18 14  6
> growth
[1]  17  20 170 315  22 190  64

> wilcox.test(gap, growth)

  Wilcoxon rank sum test with continuity correction

data:  gap and growth
W = 6.5, p-value = 0.015
alternative hypothesis: true location shift is not equal
                                                   to 0

Warning message:
In wilcox.test.default(gap, growth) :
  cannot compute exact p-value with ties
```

Warnings and Assumptions

If there are ties, the table gives approximation only.

The test does not work well if the variances are very different

> To interpret H_A as simply $\mu_1 \neq \mu_2$ rather than 'the 2 distributions are different', we actually need to assume that when the 2 distributions differ, they only differ by their means –not variances.

The Mann-Whitney test is less powerful than the t-test (when both are applicable) for small sample sizes, but almost as powerful for last sample sizes.

> Try transformation + t-test first: more powerful if applicable
>
> Otherwise use Mann-Whitney.