Comment on “Inverting the Pascal Matrix Plus One”

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The inverse of “Pascal’s matrix plus the identity”, discussed in the April 2002 issue of the Monthly, can also be presented in terms of the Bernoulli numbers $B(n)$: for all $m, n \geq 0$, the (m, n) entry of the inverse is $\binom{m}{n}\frac{1-2^{m-n+1}}{m-n+1}B(m-n+1)$. This is equivalent to a series of identities, all of which reduce to the recursion formula for the Bernoulli numbers

$$B(1) = -\frac{1}{2}, \qquad 2(2^m - 1)B(m) = -\sum_{k=1}^{m-1}\binom{m}{k}(2^k - 1)B(k) \quad m \geq 2.$$

One way to establish this recursion is to multiply the identity $D_x \log(\cos(\frac{ix}{2})) = \frac{1}{2} - \frac{1}{e^x+1}$ by $e^x + 1$ and then equate coefficients using the known expansion [Comtet, *Advanced Combinatorics*, Ex. 36, p. 88] for $\log(\cos x)$.