

Near-Simultaneous Plane Crashes

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Recently, two planes crashed in Russia 3 minutes apart. Terrorism was immediately suspected and later confirmed. What is the a priori probability of such an occurrence by chance? Assuming plane crashes obey a Poisson process averaging 80 crashes per year, the probability of two crashes occurring within 3 minutes of one another over a 50 year time span is about 5/6. This is large enough that you would have to consider it an instance of the “birthday problem” effect. Of course, the two planes took off from the same airport, sharply reducing the probability of chance occurrence. Assuming 160 equally busy airports in the world and again taking a 50 year time span, the probability of an occurrence of two planes taking off from the same airport and crashing within 3 minutes of one another is about 1.1%. Even this figure is not quite small enough to qualify as “highly significant”!

To establish these results, recall that for a Poisson process averaging α incidents or “arrivals” per unit time, the random variable $X =$ number of arrivals in a fixed time span of length t is Poisson with mean $\lambda = \alpha t$, and so

$$P(X = k) = e^{-\alpha t} \frac{(\alpha t)^k}{k!}.$$

The probability that k arrivals occur AND all the interarrival times are ≥ 1 turns out to be

$$P(X = k \text{ and minimum inter-arrival time } \geq 1) = e^{-\alpha t} \frac{\alpha^k (t - (k - 1))^k}{k!}.$$

You could think of the $e^{-\alpha t}$ as a normalizing factor that stays unchanged and otherwise the restriction that all $k - 1$ inter-arrival times be at least 1 reduces the effective time span from t to $t - (k - 1)$. Thus, the probability that the minimum inter-arrival time exceeds 1 is

$$e^{-\alpha t} \sum_{k=0}^{\lfloor t \rfloor + 1} \frac{\alpha^k (t - (k - 1))^k}{k!}.$$

With time unit 3 minutes and time span 50 years, we have $\alpha = 80/(365 * 24 * 20) = 0.000456621$ and $t = 50 * 365 * 24 * 20 = 8760000$. The preceding sum thus has over 8 million terms but they're all negligible except those in the vicinity of $k = \alpha t = 4000$. In fact, the summation range $k = \alpha t \pm 2\sqrt{\alpha t}$ gives excellent accuracy and even the one-term averaging formula— $2.5 \times (\text{summand with } k = \alpha t) \times \sqrt{\alpha t}$ —gives the first one or two significant digits over a fairly wide range of α and t . The sum is 0.16118 to 5 digits and the probability of 2 crashes within 3 minutes = $P(\text{minimum inter-arrival time} < 1) = 1 - 0.16118 = 0.83882 \approx 5/6$. For the second problem, α is slashed by a factor of 160 giving sum 0.9999286558 and for any one airport, $P(2 \text{ crashes within 3 minutes}) = 1 - 0.9999286558 = 0.0000713442$. Summing over all airports (close enough) gives $160 * 0.0000713442 \approx 1.1\%$.