Problem B3 on the 2002 Putnam Competition reads:

Show that, for $n > 1$,

$$\frac{1}{2ne} < \frac{1}{e} - \left(1 - \frac{1}{n}\right)^n < \frac{1}{ne}.$$

The most popular solution among contestants who solved the problem was neither the one published in Mathematics Magazine (February 2003) nor the one in the American Mathematical Monthly (October 2003), but rather the following:

$$\begin{array}{rccccc}
 & \frac{1}{2ne} & < & \frac{1}{e} - (1-\frac{1}{n})^n & < & \frac{1}{ne} \\
\overset{\text{rearrange}}{\Leftrightarrow} & \frac{1}{e}(1-\frac{1}{2n}) & > & (1-\frac{1}{n})^n & > & \frac{1}{e}(1-\frac{1}{n}) \\
\overset{\text{multiply by } e}{\Leftrightarrow} & 1-\frac{1}{2n} & > & e(1-\frac{1}{n})^n & > & 1-\frac{1}{n} \\
\overset{\text{take logs}}{\Leftrightarrow} & \log(1-\frac{1}{2n}) & > & 1 + n\log(1-\frac{1}{n}) & > & \log(1-\frac{1}{n}) \\
\overset{\text{Maclaurin series}}{\Leftrightarrow} & \sum_{k\geq 1} \frac{1}{k(2n)^k} & < & n\sum_{k\geq 2} \frac{1}{kn^k} & < & \sum_{k\geq 1} \frac{1}{kn^k} \\
\overset{\text{collect terms}}{\Leftrightarrow} & \sum_{k\geq 1} \frac{1}{k2^k}\frac{1}{n^k} & < & \sum_{k\geq 1} \frac{1}{k+1}\frac{1}{n^k} & < & \sum_{k\geq 1} \frac{1}{k}\frac{1}{n^k}.
\end{array}$$

The last line holds since $\frac{1}{k2^k} \leq \frac{1}{k+1} < \frac{1}{k}$ for $k \geq 1$ (comparing coefficients of $\frac{1}{n^k}$).