Journal of Computational Mathematics, Vol.12, No.2, 1994, 163–172.

MONOTONE PIECEWISE CURVE FITTING ALGORITHMS*1)

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Abstract

A piecewise cubic curve fitting algorithm preserving monotonicity of the data without modification of the assigned slopes is proposed. The algorithm has the same order of convergence as Yan's algorithm[8] and Gasparo-Morandi's algorithm[5] for accurate or $O(h^q)$ accurate given data, but it has a more visually pleasing curve than those two algorithms. We also discuss the convergence order of cubic rational interpolation for $O(h^q)$ accurate data.

1. Introduction

The problem of monotonicity preserved interpolation has been considered by a number of authors. Fritsch and Carlson[4] have obtained necessary and sufficient conditions for a cubic Hermite interpolant to be monotone on an interval. Eisentat, Jackson and Lewis[3] derived a fourth-order accurate algorithm which is a modification of Fritsch and Carlson's algorithm. Beatson and Wolkowicz[1] considered monotone interpolation schemes of the fitting and modifying type, and gave the optimal order error properties of their algorithms. Gregory and Delbourgo[6] gave an explicit representation of a piecewise rational quadratic function; they also gave an explicit representation of a piecewise rational cubic function[2]; both explicit representations produce monotone interpolation for given monotone data. Yan[8] gave a piecewise cubic curve fitting algorithm without modification of the assigned slopes through inserting two knots to construct a horizontal line on a non-monotone interval. Gasparo-Morandi's algorithm[5] is a modification of Yan's algorithm[8], which inserts two knots to construct a slope line on a non-monotone interval.

Our algorithm which inserts two knots to construct two quadratic curves on a non-monotone interval is also a modification of Yan's algorithm[8] and Gasparo-Morandi's algorithm[5]. An $O(h^4)$ convergence result is obtained when the exact function and derivative values are available; otherwise, an $O(h^p)$ $(p = \min(4, q))$ convergence is obtained for an $O(h^q)$ accurate function and derivative values. The proof process of the main result is similiar to that in Yan[8] and Gasparo-Morandi[5]. We also discuss

* Received June 3, 1993.
1) The Project Supported by National Natural Science Foundation of China.

the convergence order of cubic rational interpolation with the $O(h^q)$ accurate function and derivative values, and an $O(h^p)$ convergence is obtained.

The paper begins with a definition of cubic interpolant, necessary and sufficient conditions, and construction of our algorithm. The convergence analysis of the algorithm is discussed in **3**. The convergence analysis of cubic rational interpolation for $O(h^q)$ accurate data is discussed in **4**. Finally, in **5**, examples applied with various interpolation methods and comparison are given.

2. The Algorithm

Let $f(x) \in C^1[a,b]$ be a monotone increasing function. Let $\pi : a = x_1 < x_2 < \cdots < x_n = b$ be a partition of the interval $I = [a,b]$. Suppose that y_i and d_i are approximate values of $f(x)$ and $f'(x)$ at the partition points x_i respectively. Let $h_i = x_{i+1} - x_i, \Delta y_i = y_{i+1} - y_i, \Delta_i = \Delta y_i / h_i, i = 1, 2, \cdots, n$. In particular we suppose that there exists an integer $q > 0$ such that

$$y_i = f(x_i) + O(h^q), \qquad d_i = f'(x_i) + O(h^q), \qquad i = 1, 2, \cdots, n \tag{2.1}$$

where $h = \max\{h_i\}$. Now, we construct a piecewise cubic function $s(x) \in C^1[I]$ such that

$$s(x_i) = y_i, \qquad s'(x_i) = d_i, \qquad i = 1, 2, \cdots, n. \tag{2.2}$$

In each subinterval $I_i = [x_i, x_{i+1}]$, s(x) is defined by

$$s_i(x) = \frac{d_i + d_{i+1} - 2\Delta_i}{h_i^2}(x - x_i)^3 + \frac{-2d_i - d_{i+1} + 3\Delta_i}{h_i}(x - x_i)^2 + d_i(x - x_i) + y_i. \tag{2.3}$$

It is clear that a necessary condition for monotonicity is that

$$\operatorname{sgn}(d_i) = \operatorname{sgn}(d_{i+1}) = \operatorname{sgn}(\Delta_i). \tag{2.4}$$

Furthermore, if $\Delta_i = 0$, then $s(x)$ is monotone (i.e. constant) on I_i if and only if $d_i = d_{i+1} = 0$. The remainder of this section assumes that $\Delta_i \neq 0$ and (2.4) is satisfied. Let $\alpha_i = d_i/\Delta_i, \beta_i = d_{i+1}/\Delta_i$. Then we have the following lemmas[4].

Lemma 1. *If $\alpha_i + \beta_i - 2 \le 0$, then $s(x)$ is monotone on I_i if and only if* (2.4) *is satisfied.*

Lemma 2. *If $\alpha_i + \beta_i - 2 > 0$, and* (2.4) *is satisfied, then $s(x)$ is monotone on I_i if and only if one of the following conditions is satisfied*:

(i) $2\alpha_i + \beta_i + 3 \le 0$,

(ii) $\alpha_i + 2\beta_i - 3 \le 0$, *or*

(iii) $\phi(\alpha_i, \beta_i) \ge 0$,

where $\phi(\alpha, \beta) = \alpha - (2\alpha + \beta - 3)^2 / 3(\alpha + \beta - 2)$.

In general, $s_i'(x)$ has the following form: $s_i'(x) = a(x - \bar{x})^2 + \omega$, where $\bar{x}$ is the extreme point of $s_i'(x)$. We denote μ, η, ω as $\mu = \bar{x} - x_i$, $\eta = x_{i+1} - \bar{x}$, $\omega = s_i'(\bar{x})$. It is clear that $s(x)$ is not monotone on I_i if and only if

$$0 < \mu, \quad \eta < h_i \quad \text{and} \quad \Delta_i \omega < 0. \tag{2.5}$$

Let $c_1 < \bar{x} < c_2$ be three inserting points on subinterval I_i. Let $g_1(x) = (\bar{x}-x)/(\bar{x}-c_1)$, $g_2(x) = (\bar{x}-x)/(\bar{x}-c_2)$. A new interpolant $\tilde{s}(x)$ on I_i will be taken to have a derivative $\tilde{s}'(x)$ of the following form:

$$\tilde{s}_i'(x) = \begin{cases} a_1(x-c_1)^2 + c, & x \in [x_i, c_1], \\ cg_1(x), & x \in [c_1, \bar{x}], \\ cg_2(x), & x \in [\bar{x}, c_2], \\ a_2(x-c_2)^2 + c, & x \in [c_2, x_{i+1}] \end{cases} \tag{2.6}$$

where $\bar{x}$ is the extreme point of $s'(x)$ on I_i, and the constants a_1, a_2 and c and the additional points c_1 and c_2 must be determined in such a way that $\tilde{s}(x)$ satisfies all interpolation and monotonicity requirments. Two special cases of $\tilde{s}'(x)$ are given by Yan with $g_1(x) = 0$, $g_2(x) = 0$ and by Gasparo and Morandi with $g_1(x) = 1$, $g_2(x) = 1$. From (2.6), we must choose c such that $c\Delta_i > 0$. Let

$$\tilde{\mu} = c_1 - x_i \qquad \text{and} \qquad \tilde{\eta} = x_{i+1} - c_2. \tag{2.7}$$

The derivative interpolation conditions can be expressed as

$$a_1\tilde{\mu}^2 + c = d_i \qquad \text{and} \qquad a_2\tilde{\eta}^2 + c = d_{i+1}. \tag{2.8}$$

Based on the derivative interpolation conditions, we have actually infinitely many choices to determine a_1, a_2, c_1, c_2, c. By integrating $\tilde{s}_i'(x)$ and $s_i'(x)$ on $[x_i, x_{i+1}]$, and letting they have the same integrating value, we obtain the folowing equation:

$$\tilde{\mu}d_i + \tilde{\eta}d_{i+1} + \frac{c}{2}(3h_i + \mu + \eta) = 3\Delta y_i. \tag{2.9}$$

By letting

$$\frac{\tilde{\mu}}{\tilde{\eta}} = \frac{\mu}{\eta} \tag{2.10}$$

we obtain two equations that have three unknown variables. Let c be a free variable. We can determine μ and η (i.e. a_1, a_2, c_1, c_2) from (2.9) and (2.10). In fact, the linear system (2.9)–(2.10) has a unique solution if and only if its determinant $D_i = -(d_i\mu + d_{i+1}\eta + ch_i/2) \neq 0$. In this case we obtain the following solution:

$$\tilde{\mu} = \rho(c)\mu \qquad \text{and} \qquad \tilde{\eta} = \rho(c)\eta \tag{2.11}$$

where $\rho(c) = 3(\Delta_i - \frac{c}{2})/(\theta + \frac{c}{2})$, and $\theta = (d_i\mu + d_{i+1}\eta)/h_i$. In order to ensure that $x_i < c_1 \leq \bar{x} \leq c_2 < x_{i+1}$, we must determine c such that $0 < \rho(c) \leq 1$ and $c\Delta_i \geq 0$. For this purpose, by integrating $s_i'(x)$ on I_i and using interpolation properties we obtain the relation

$$d_i\mu + d_{i+1}\eta + 2\omega h_i = 3\Delta y_i. \tag{2.12}$$

For increasing data, we have $\omega < 0 < \Delta_i < 2\Delta_i < 3\Delta_i < \theta$ and $\rho(c)$ is a monotone decreasing function such that $0 < \rho(c) \leq 1$ for $c \in [0, 2\Delta_i]$. For decreasing data, we also obtain $0 < \rho(c) \leq 1$ for $c \in [2\Delta_i, 0]$.

In conclusion, $\tilde{s}'(x)$ can be determined in the following way. We choose a value of c in the interval $[0, 2\Delta_i]$ or $[2\Delta_i, 0]$ for increasing or decreasing data respectively. Then we can determine a_1, a_2, c_1, c_2 from (2.6) and (2.7) and $\tilde{s}_i'(x)$ is obtained. Finally, by integrating $\tilde{s}_i'(x)$ on I_i, $\tilde{s}_i(x)$ can be obtained.

3. The Convergence Order of Cubic Interpolant

We shall show that the algorithm described in Section 2 gives an $O(h^p)$ convergence order. We discuss only the case when $f(x)$ is nondecreasing. Let $\|\cdot\|$ denote the uniform norm on I. Let $m(x)$ be defined by

$$m(x) = \begin{cases} s_i(x), & s_i(x) \text{ is monotone on } I_i, \\ \tilde{s}_i(x), & \text{otherwise} \end{cases} \tag{3.1}$$

for $x \in [x_i, x_{i+1}]$, $\quad i = 0, 1, \cdots, n$. We have the following convergence theorem.

Theorem 3.1. *Let f(x)$\in C^4[a,b]$ and $p = \min(4, q)$. If* (2.1) *holds and the variable c in $\tilde{s}_i(x)$ satisfies*

$$0 \leq c < \min(|\,\omega\,|, 2\Delta_i), \tag{3.2}$$

then for each interval I_i, we have $\| f(x) - m(x) \| = O(h^p)$.

Proof. Let $t(x)$ be a cubic interpolant for exact $f(x_i)$, $f'(x_i)$ on I_i. It is well-known[8] that

$$\| f - t \| = O(h^4), \qquad \| f' - t' \| = O(h^3). \tag{3.3}$$

Note that both $t(x)$ and $s_i(x)$ are cubic polynomials. By using the triangular inequality we have

$$\| f - s_i \| = O(h^p), \qquad \| f' - s_i' \| = O(h^{p-1}). \tag{3.4}$$

If $m(x) \equiv s_i(x)$ on I_i, we complete our proof. Now let $m(x) \equiv \tilde{s}_i(x)$ on I_i, we have

$$\| f - m \| \equiv \| f - \tilde{s}_i \| \leq \| f - s_i \| + \| s_i - \tilde{s}_i \| . \tag{3.5}$$

From (3.4), we need to estimate $\| s_i - \tilde{s}_i \|$. For this purpose, we have

$$\| s_i - \tilde{s}_i \| \leq \int_{x_i}^{x} \| s_i' - \tilde{s}_i' \| \, dy \leq \int_{x_i}^{x_{i+1}} \| s_i' - \tilde{s}_i' \| \, dy \leq h_i \| s_i' - \tilde{s}_i' \| . \tag{3.6}$$

Let us find the super value of $|\, s_i'(x) - \tilde{s}_i'(x) \,|$ on $[x_i, \bar{x}]$. (For the same reason, we can find the super value of $|\, s_i'(x) - \tilde{s}_i'(x) \,|$ on $[\bar{x}, x_{i+1}]$). Let $D(x)$ be defined by $D(x) = s_i'(x) - \tilde{s}_i'(x)$, $\quad x \in [x_i, \bar{x}]$, because $s_i'(x)$ can be expressed as $s_i'(x) = a(x - \bar{x})^2 + \omega$, where $a \in \Re$ such that

$$a\mu^2 + \omega = d_i. \tag{3.7}$$

$D(x)$ has the following form

$$D(x) = \begin{cases} a(x - \bar{x})^2 + \omega - a_1(x - c_1)^2 - c, & x \in [x_i, c_1], \\ a(x - \bar{x})^2 + \omega - c g_1(x), & x \in [c_1, \bar{x}]. \end{cases} \tag{3.8}$$

Firstly, we consider the values of $D(x)$ on $[c_1, \bar{x}]$. From (3.8), we know that $D(x) \in C[x_i, \bar{x}]$. Then $D(x)$ has its extreme values on $[x_i, \bar{x}]$. Because $\omega < 0$, (3.7) means

$a > 0$. Let $\tilde{D}(x) = a(x-\bar{x})^2 + |\omega| + cg_1(x)$. $|D(x)| \le \tilde{D}(x)$. $\tilde{D}(x)$ has an inf value on $\Re$. Let $\tilde{D}'(x) = 0$. We obtain the extreme point of $\tilde{D}(x)$ as $\tilde{x} = \bar{x} + c/(2a(\bar{x} - c_1))$. Because $\tilde{x} > \bar{x}$, $\tilde{x}$ is located on the right side of subinterval $[c_1, \bar{x}]$. $\tilde{D}(x)$ is monotone decreasing on $[c_1, \bar{x}]$. Its extreme points are the end-points of subinterval $[c_1, \bar{x}]$. For right end-point $\bar{x}$, we have

$$D(\bar{x}) = |\omega|. \tag{3.9}$$

The estimation of $|D(c_1)|$ will be discussed after the estimation of the supervalue of $|D(x)|$ on subinterval $[x_i, c_1]$ is discussed.

Because $D(x_i) = 0$, we assume that the maximum value of $|D(x)|$ on $[x_i, c_1]$ is at x_0, where $x_0 \in (x_i, c_1)$. We have the following relation:

$$a(x_0 - \bar{x}) - a_1(x_0 - c_1) = 0. \tag{3.10}$$

From (3.7) and (3.10), we have

$$|D(x_0)| \le |\omega| + c + (d_i + |\omega|)\,|\bar{x} - c_1|/\mu. \tag{3.11}$$

It is obvious that $0 < \frac{\bar{x}-c_1}{\mu} < 1$. We have

$$\frac{\bar{x} - c_1}{\mu} = \frac{-2\omega h_i + 2ch_i}{d_i\mu + d_{i+1}\eta + ch_i/2} < \frac{(2c - 2\omega)h_i}{d_i\mu + d_{i+1}\eta}. \tag{3.12}$$

If $0 \le d_i \le d_{i+1}$, we have

$$\frac{h_i}{d_i\mu + d_{i+1}\eta} \le \frac{h_i}{d_i(\mu+\eta)} = \frac{1}{d_i}. \tag{3.13}$$

If $d_i \ge d_{i+1} \ge 0$, we have $0 \le \eta \le \frac{h_i}{2} \le \mu$ and

$$\frac{h_i}{d_i\mu + d_{i+1}\eta} \le \frac{2\mu}{d_i\mu} = \frac{2}{d_i}. \tag{3.14}$$

From (3.13) and (3.14) we have

$$\frac{|\bar{x} - c_1|}{\mu} \le \frac{2\,|2c - 2\omega|}{d_i} \le \frac{4c + 4|\omega|}{d_i}. \tag{3.15}$$

From (3.12) and (3.15) we have

$$|D(x_0)| \le 5|\omega| + 5c + 4|\omega|(|\omega| + c)/d_i. \tag{3.16}$$

If $d_i \ge |\omega| + c$, we have

$$|D(x_0)| \le 9|\omega| + 5c \le 9(|\omega| + c). \tag{3.17}$$

If $d_i \le |\omega| + c$, we have

$$|D(x)| \le |\tilde{s}_i'(x)| + |s_i'(x)| \le \max(c, d_i) + d_i \le 2(|\omega| + c). \tag{3.18}$$

From (3.16) and (3.18) we have

$$\| s_i' - \tilde{s}_i' \| \le 9(|\omega| + c), \quad x \in [x_i, c_1]. \tag{3.19}$$

Now, we discuss the estimation of $|D(c_1)|$. From (3.15) we have

$$|D(c_1)| \le |\omega| + c + 16((|\omega| + c)^2/d_i + |\omega|(((|\omega| + c)/d_i)^2). \tag{3.20}$$

If $d_i \geq |\omega| + c$, we have

$$|D(c_1)| \leq |\omega| + c + 16(2|\omega| + c) \leq 33(|\omega| + c). \tag{3.21}$$

If $d_i \leq |\omega| + c$, we have

$$|D(c_1)| \leq |\tilde{s}_i'(c_1)| + |s_i'(c_1)| \leq 2(|\omega| + c). \tag{3.22}$$

From (3.9), (3.19) and (3.22) we have

$$|D(x)| \leq 33(|\omega| + c) \qquad x \in [x_i, \bar{x}]. \tag{3.23}$$

We have $|\omega| = |s_i'(\bar{x})| \leq |s_i'(x) - f'(x)| \leq \| s_i' - f' \| = O(h^{p-1})$ and $c < |\omega| \leq O(h^{p-1})$. From (3.23), (3.6),(3.5),(3.4) and the definition of $D(x)$ we obtain $\| s_i' - \tilde{s}_i' \| = O(h^{p-1})$, $\| s_i - \tilde{s}_i \| = O(h^p)$ and complete the proof of Theorem 3.1.

4. The Convergence Analysis of Cubic Rational Interpolation for an $O(h^q)$ Cccurate Function and Derivative Values

We take the code of Section 3 and assumptions (2.1) and (2.4). Furthermore, let parameter $r_i > -1$. A piecewise cubic rational interpolation $z(x)$ for the exact function and derivative values on I_i is defined by

$$z(x) = \frac{p(\theta)}{q(\theta)} \quad x \in [x_i, x_{i+1}] \tag{4.1}$$

where $\theta = (x - x_i)/h_i$, $p(\theta) = f_{i+1}\theta^3 + (r_i f_{i+1} - h_i f_{i+1}')\theta^2(1-\theta) + (r_i f_i + h_i f_i')\theta(1-\theta)^2 + f_i(1-\theta)^3$, $q(\theta) = 1 + (r_i - 3)\theta(1-\theta)$. A piecewise cubic rational interpolation $\tilde{z}(x)$ for the $O(h^q)$ accurate function and derivative values on I_i is defined by

$$\tilde{z}(x) = \frac{\tilde{p}(\theta)}{q(\theta)} \quad x \in [x_i, x_{i+1}] \tag{4.2}$$

where $\tilde{p}(\theta) = y_{i+1}\theta^3 + (r_i y_{i+1} - h_i d_{i+1})\theta^2(1-\theta) + (r_i y_i + h_i d_i)\theta(1-\theta)^2 + y_i(1-\theta)^3$. Furthermore $z(x)$ and $\tilde{z}(x)$ have the following interpolation properties:

$$z(x_i) = f_i, \qquad z(x_{i+1}) = f_{i+1}, \qquad z'(x_i) = f_i', \qquad z'(x_{i+1}) = f_{i+1}' \tag{4.3}$$

$$\tilde{z}(x_i) = y_i, \qquad \tilde{z}(x_{i+1}) = y_{i+1}, \qquad \tilde{z}'(x_i) = d_i, \qquad \tilde{z}'(x_{i+1}) = d_{i+1}. \tag{4.4}$$

When $r_i \geq \dfrac{d_i + d_{i+1}}{\Delta_i}$, $z(x)$ is monotone increasing[2]. We observe that the cubic rational interpolation $z(x)$ and $\tilde{z}(x)$ will degenerate into cubic interpolants $s(x)$ and $\tilde{s}(x)$ when $r_i = 3$. In the remainder of this section we discuss the limit behavior of $f(x) - \tilde{z}(x)$ when $h \to 0$. Firstly, we give a lemma from [6]. Then we introduce our two lemmas.

Lemma 4.1 *Let $f(x) \in C^4[a,b]$. If (4.3) holds, then*

$$|f(x) - z(x)| \leq \{h_i^4 \| f^{(4)} \| (1 + |r_i - 3|/4) + 4|r_i - 3|(h_i^3 \| f^{(3)} \| + 3h_i^2 \| f^{(2)} \|)\}/384\theta_i \quad x \in [x_i, x_{i+1}] \tag{4.5}$$

where

$$\theta_i = \begin{cases} (1+r_i)/4, & \text{if } -1 < r_i < 3, \\ 1, & \text{if } r_i \geq 3. \end{cases}$$

Lemma 4.2. *Under the assumption of Lemma* 4.1, *if* $r_i - 3 = O(h_i^l)$, *then* $\| f - z \| = O(h_i^m)$, *where* $m = \min(4, l+2)$.

Lemma 4.3 *If (2.1) holds and* $r_i - 3 = O(h_i^l)$, *then* $\| z - \tilde{z} \| = O(h^q)$.

Proof. we can express (2.1) as

$$y_i = f_i + e_i, \qquad e_i = O(h^q), \qquad d_i = f_i' + e_i', \qquad e_i' = O(h^q). \tag{4.6}$$

Then,

$$| z(x) - \tilde{z}(x) | \leq \{ | e_{i+1} | + 2 | r_i || e_{i+1} | /9 + 2h_i | e_{i+1}' | /9 + | r_i || e_i | /8$$
$$+ h_i | e_i' | /8 + | e_i | \} / q_i(\theta). \tag{4.7}$$

Finally,

$$| q_i(\theta) | \geq \begin{cases} 1 - (3 - r_i)/4, & \text{if } -1 < r_i < 3, \\ 1, & \text{if } r_i \geq 3. \end{cases} \tag{4.8}$$

Combining (4.6),(4.7) and (4.8) complete the proof of the lemma.

From Lemma 4.2 and Lemma 4.3 we have

Theorem 4.1. *Let* $r_i - 3 = O(h_i^l)$. *If* (2.1) *holds, then*

$$\| f - \tilde{z} \| = O(h^p), \tag{4.9}$$

where $p = \min(4, q, l+2)$.

A direct consequence of Theorem 4.1 which is similiar to Theorem 3.1 is the following corollary.

Corollary 4.1. *Let* $r_i - 3 = O(h_i^2)$. *If* (2.1) *holds, then*

$$\| f - \tilde{z} \| = O(h^p), \tag{4.10}$$

where $p = \min(4, q)$.

To ensure that $r_i - 3 = O(h_i^2)$, a suggested selection is $r_i = 1 + \dfrac{d_i + d_{i+1}}{\Delta_i}$ in [6].

5. Numerical Examples and Conclusion

In this section, we compare the results of the method described in Section 2 with those of Yan's algorithm, Gasparo-Morandi's algorithm and cubic rational interpolation for two of the typical data sets considered in the literature. The derivative values are approximated by a four point formula in [8]. In order to satisfy (3.2) we choose $c = 0.95 \min(| \omega |, 2\Delta_i)$ on non-monotone subintervals. Figures are drawn by GS system[9] on IBM4341 computer.

The first data set, used in Figs. 1–4, is from Akima in [4], namely

x	0	2	3	5	6	8	9	11	12	14	15
y	10	10	10	10	10	10	10.5	15	50	60	85

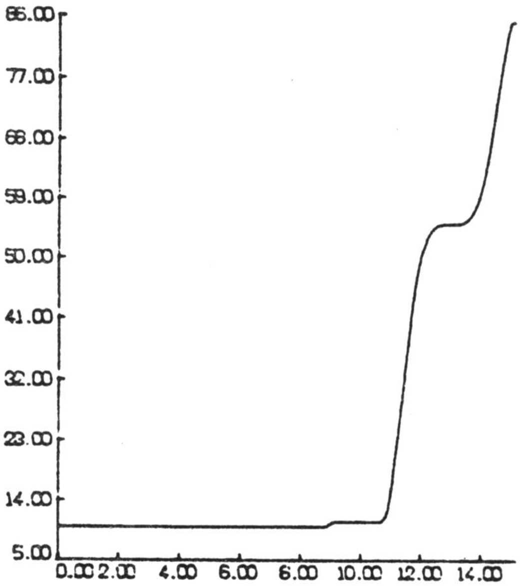

Fig.1. Yan's algorithm

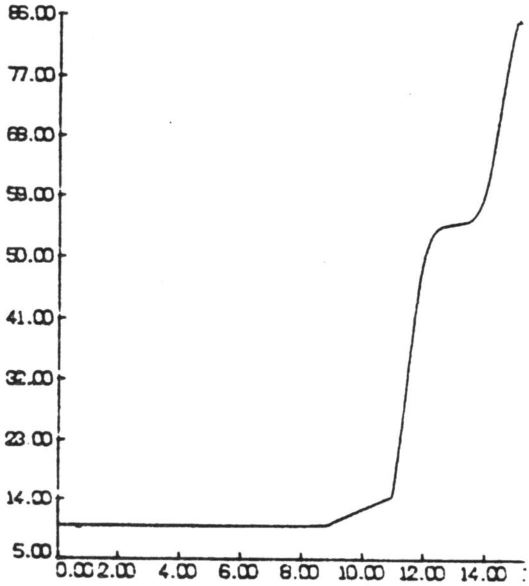

Fig.2. Gasparo-Morandi's algorithm

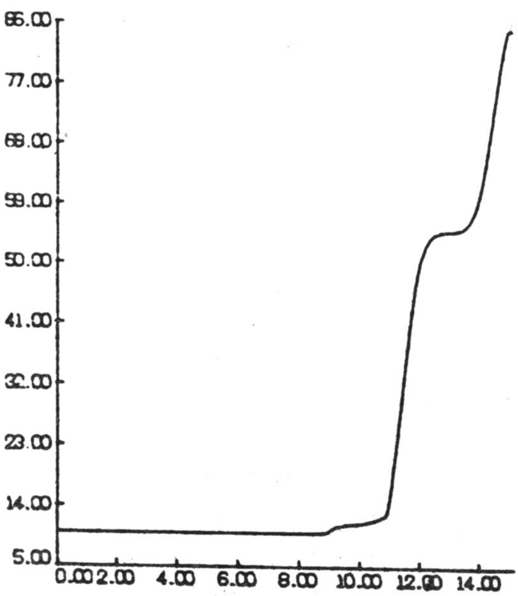

Fig.3. The algorithm

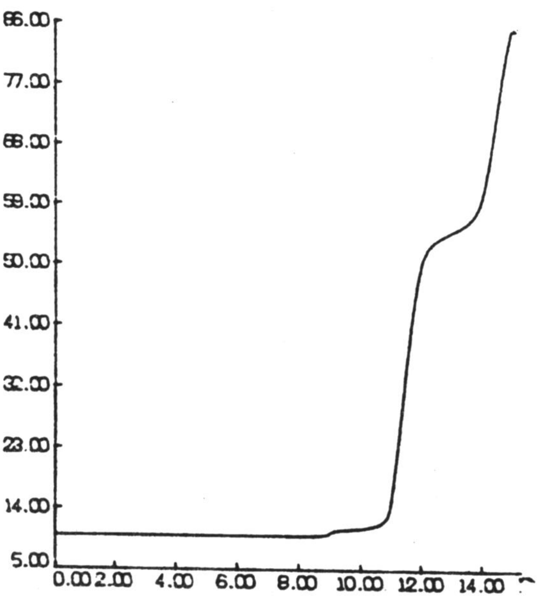

Fig.4. Cubic rational interpolation

The second data set, used in Figs. 5–8, is from RPN14 in [4], namely

x	7.99	8.09	8.19	8.7	9.2
y	0	2.76429E-5	4.37498E-2	0.169183	0.469428

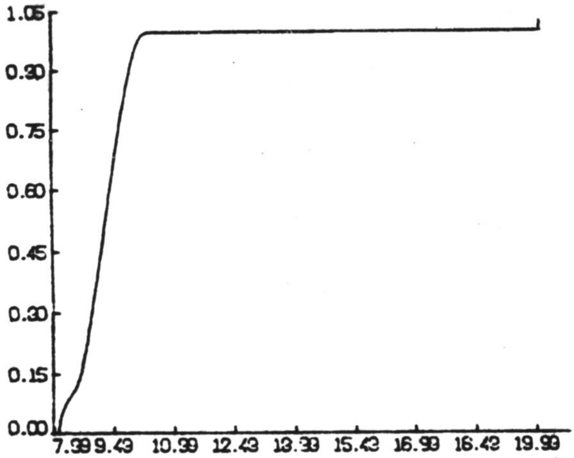

Fig.5. Yan's algorithm

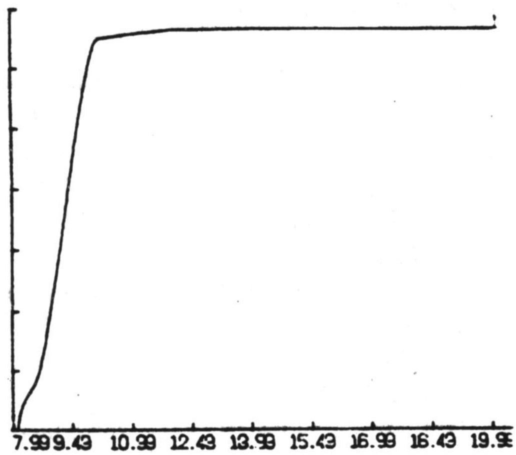

Fig.6. Gasparo-Morandi's algorithm

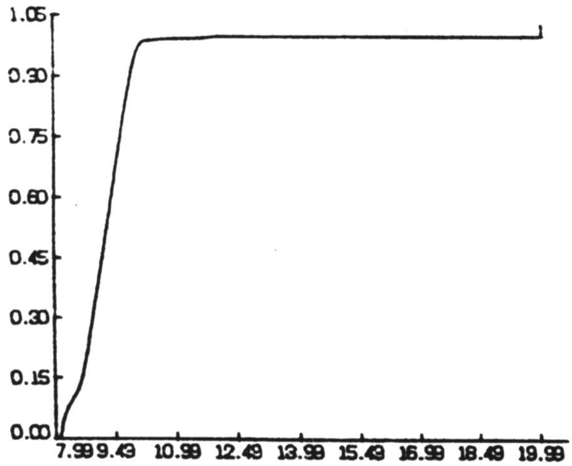

Fig.7. The algorithm

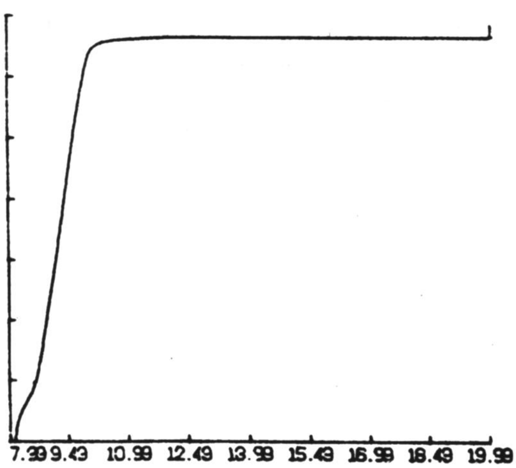

Fig.8. Cubic rational interpolation

x	10	12	15	20
y	0.943740	0.998636	0.999919	0.999994

For those two typical data sets, we see that both the algorithm and cubic rational interpolation produce visually pleasing curves. When the slopes of the data change abruptly, the graphs of the algorithm are similiar to cubic rational interpolation, but the graphs produced by other methods are not. From this character and the good error bounds, we can say the algorithm is better. A successful application of the algorithm for variate generation has been given in [10].

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