Lecture 1

Population and Sample

Lecture Summary

- We have a **population** to conduct our study.
- Often, we can’t gather information from every member of the population. Therefore, we **sample!**
- From the sample, we investigate various features of the population, called **parameters**
- We do this by creating **statistics** based on the sample

Population

- **Population**: A collection of objects for study

- Example 1:
  - Goal: Study the efficacy of a new malaria vaccine
  - Population: Individuals prone to malarial infection
  - Why not just have all individuals as the population?

- Example 2:
  - Goal: Study the pattern of spam mail in Gmail
  - Population: All the possible spam mail that are (and will be in Google's servers)
  - Note: objects in the population may not exist!

See any Patterns?

Weekend

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"It's Friday, Friday, Gotta get down on Friday…"

Sample

- Often, we can’t take measurements for every single object in the population
  - Expensive, morally unjustified, etc.
  - May not even exist yet!

- **Sample**: A manageable subset of the population that is representative of the population
  - Size of subset denoted as n
  - Measurements from sample denoted as $X_1, \dots, X_n$

Parameters

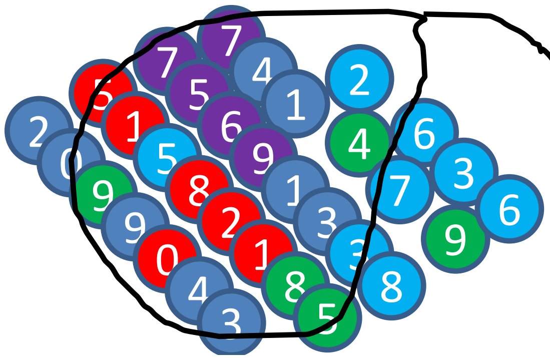
- **Parameters**: numerical features/descriptions/characteristics of the population, usually unknown
  - From example 1 (malaria vaccine efficacy):
    - Distribution of body temperature for all individuals after vaccination
    - Average difference in parasite levels for all individuals before and after vaccination
  - From example 2 (Gmail spam pattern):
    - Average word count in spam
    - Frequency of spam for each day of the week

Statistic

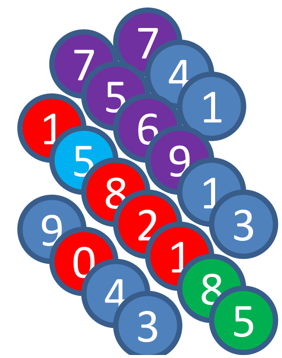
- **Statistic**: a function of the sample that is used to estimate/infer about the unknown **parameters**!
  - Examples: Sample mean, sample variance, empirical distribution/frequency, etc.

- Generally a statistic is denoted as $T(X_1, \dots, X_n)$ or T where T is a function of the sample

Population/Parameter and Sample/Statistic



Features of the population
(parameters)



Estimates of the features
(statistics)

Mean: $\mu = 4.6364$
Distribution:

Red	DBlue	LBlue	Green	Purple
6	9	8	5	5

Mean: $\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} X_i = 4.619048$
Empirical Distribution/Frequency

Red	DBlue	LBlue	Green	Purple
5	7	1	2	5

Population/Sample with Malaria

Parameter

Distribution of body temperature for all individuals after vaccination

- $F(x)$: cdf of X

Statistic

Empirical distribution of body temperature for vaccinated individuals in the sample

- $T(X_1, \ldots, X_n) = \frac{1}{n}\sum_{i=1}^{n} I(X_i \leq x)$

Average difference in parasite levels for all individuals before vaccination

- $\mu = E(X)$
- $X \sim F_\mu$, independent and identically distributed

Sample average difference in parasite levels before vaccination

- $T(X_1, \ldots X_n) = \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

How old am I?

1) What is the population
2) What is my sample
3) What parameters am I interested in
4) What statistics should I use to estimate the parameters?

Summary

- **Population**: a collection of units
  - **Parameters**: numerical description of the collection
    - E.g. Mean, variance, cumulative distribution function, etc.
- **Sample**: a manageable and representative collection of units
  - We derive **statistics** that estimate the parameters
    - E.g. Sample mean, sample variance, empirical distribution function, etc.

Extra Slides

Representative Sampling Strategies

- **Simple Random Sampling (SRS)**: randomly sample n objects from the population
  - Any n-subset of the population is equally likely
  - If objects are randomly sampled with replacement or if the population size is infinite, it is i.i.d. (independent and identically distributed…more on this later)

- **Stratified Sampling**: divide the population into K homogenous groups and perform SRS on each group
  - <u>Example 1</u>: Efficacy of malaria vaccine
  - Divide the population into children and adults.