Name:
Quiz 1, Stat 431 (Summer 2012)
July 8, 2012

1. (Population and Paramter/Sample and Statistic)

   (a) (TRUE/FALSE): Parameters are random variables

   (b) (TRUE/FALSE): The sample mean and the sample variance are random variables

   (c) Suppose we have data listed as $(X_1, ..., X_n)$. Which of the following statistics are sensitive to outliers? Circle all that apply.

      i. Sample mean, $\bar{X}$
      ii. Sample median, $X_{0.5}$
      iii. Sample variance, $\hat{\sigma}^2$
      iv. Interquartile range, $X_{0.75} - X_{0.25}$
      v. The maximum, $\max(X_1, ..., X_n)$

2. Suppose we have $(X_1, ..., X_n)$ where $X_i \overset{\text{iid}}{\sim} F(\mu, \sigma^2)$ for some arbitrary F. In elementary statistics classes, there is no mathematical justification as to why $\frac{1}{n-1}\sum_{i=1}^n (X_i - \bar{X})^2$ is used instead of $\frac{1}{n}\sum_{i=1}^n (X_i - \bar{X})^2$ in estimating the population variance, σ^2. In these series of (hopefully) short questions, we'll provide one mathematical justification as why $\frac{1}{n-1}$ is preferred over $\frac{1}{n}$

   (a) Is $\frac{1}{n-1}\sum_{i=1}^n (X_i - \bar{X})^2$ an unbiased estimator for σ^2? A simple Yes/No will suffice.

   (b) Is $\frac{1}{n}\sum_{i=1}^n (X_i - \bar{X})^2$ an unbiased estimator for σ^2? *You must provide mathematical justification for your answer.*

3. Suppose we have $(X_1, ..., X_n)$ where $X_i \overset{\text{iid}}{\sim} F(\mu, \sigma^2)$ for some arbitrary F and we know what the population mean, μ ,is. Consider the following estimators for the population variance

$$\text{Estimator 1: } \frac{1}{n-1}\sum_{i=1}^n (X_i - \mu)^2$$

$$\text{Estimator 2: } \frac{1}{n}\sum_{i=1}^n (X_i - \mu)^2$$

   (a) Which estimator is an unbiased estimate for the population variance? *You must provide mathematical justification for your answer*

(b) If F is a normal distribution, what is the sampling distribution of the unbiased estimator?
Hint: You may need some constants to be multiplied to your estimator and use the definition of Chi-square discussed in class

(c) If F is any arbitrary distribution, what is the limiting distribution of the unbiased estimator?
Hint: You may need some constants to be multiplied and added to your estimator before you use the Central Limit Theorem. The answer is NOT Chi-Squared, χ^2.