STAT 451 Midterm Exam NetID: ____________

Last name: ______________________________ First name: ________________________

Instructions:

1. Do not open the exam until I say "go."
2. Put away everything except a pencil or pen, a calculator, and your two one-page (two sides each) notes sheets.
3. Show your work. Correct answers without enough work may receive no credit.
4. If a question is ambiguous, resolve the ambiguity in writing. We will consider grading accordingly.
5. The exam ends when I call time. If you continue writing after I call time, you risk a penalty. (The alternative, that you get more time than your peers, is unfair.)
6. You are welcome to turn your exam in to me before I call time. However, if you are still here in the last five minutes, please remain seated until I've called time (to avoid disturbing peers).

Question	Points	Earned
Q0 (cover)	1	
Q1	9	
Q2	12	
Q3	6	
Q4	12	
Q5	3	
Q6	3	
Q7	4	
Total	50	

1. Consider a decision tree node containing the set of examples $S = \{(\mathbf{x}, y)\}$ where $\mathbf{x} = (x_1, x_2)$:

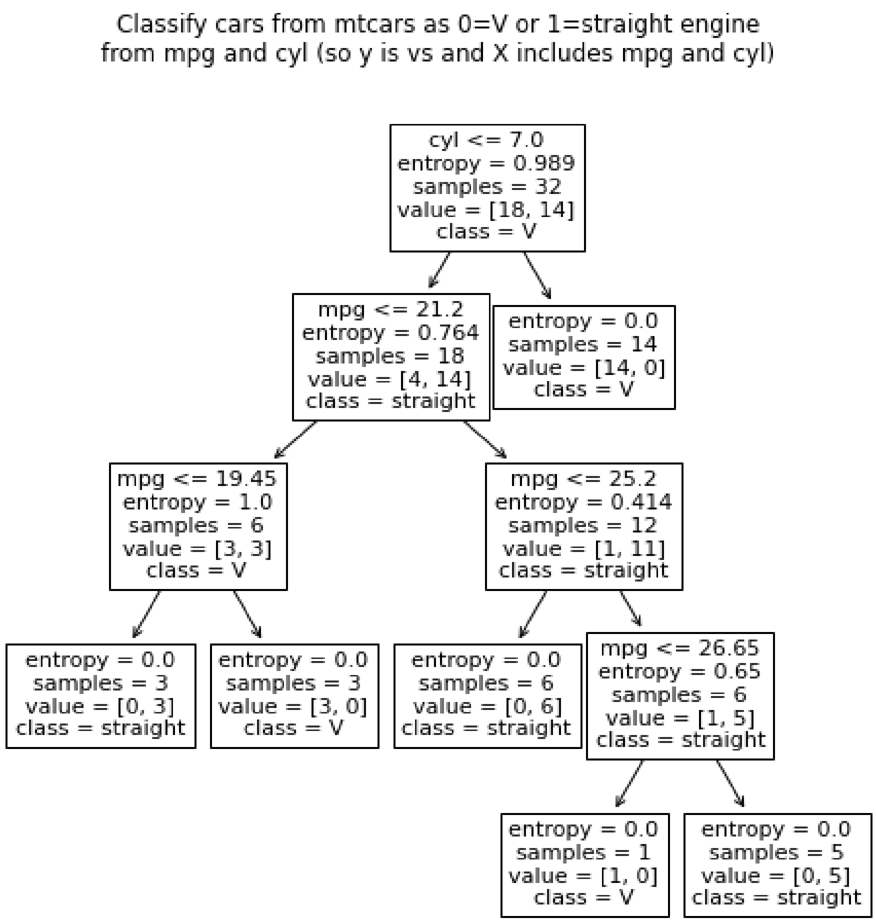
S

x_1	x_2	y
4	9	1
2	6	0
5	7	0
3	8	1

(a) The entropy of this node in bits is _______.

(b) The (feature, threshold) pair (j, t) that yields the best split for this node is feature $j =$ _______ and threshold $t =$ _______.

(c) Now consider this tree:



This tree says a car whose gas mileage (mpg) is 26 and number of engine cylinders (cyl) is 4 has a _____________ engine.

2. Mark each statement true or false by circling the appropriate choice.

   (a) TRUE / FALSE An SVM makes a classification error on $\mathbf{x}$ when $\mathbf{wx} + b \in (-1, 1)$ (i.e. between -1 and 1).

   (b) TRUE / FALSE In logistic regression, we model $P(y = 1)$ as a linear function of $\mathbf{x}$.

   (c) TRUE / FALSE In linear regression, a reasonable alternative to the typical objective function *mean squared error* $= \frac{1}{N}\sum_{i=1}^{N}[f_{\mathbf{w},b}(\mathbf{x}_i) - y_i]^2$ is *mean error* $= \frac{1}{N}\sum_{i=1}^{N}[f_{\mathbf{w},b}(\mathbf{x}_i) - y_i]$.

   (d) TRUE / FALSE In a decision tree node, an entropy of 1 indicates all the node's examples have the same y value.

   (e) TRUE/ FALSE For training data $\{(\mathbf{x}, y)\}$ such that $\mathbf{x}_i \neq \mathbf{x}_j$ for all i and j, we can build a kNN model that classifies the training examples without error.

   (f) TRUE / FALSE If we train an SVM on linearly separable data, then discard all training examples which are not support vectors, and then train a new SVM on the remaining examples, the first SVM will classify unseen examples better than the second.

3. Here are two questions about feature engineering.

   (a) Use one-hot encoding to transform the categorical feature `weather` into binary features with reasonable names.

(input) `weather`	(output)
sunny	
raining	
cloudy	
raining	

   (b) Do min-max rescaling on feature `x`:

(input) `x`	(output) `x_rescaled`
3	
1	
2	

4. Consider the logistic regression model,

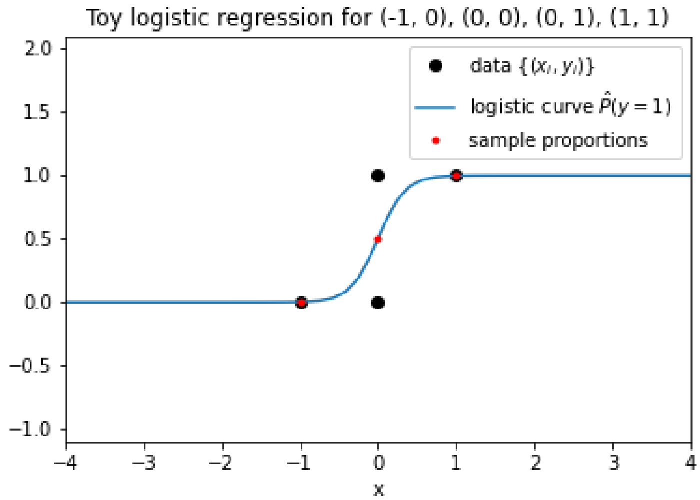
$$P(y_i = 1) = \frac{1}{1 + e^{-(\mathbf{wx}+b)}} .$$

(a) Logistic regression is named after the log-odds of success, $\ln \frac{p}{1-p}$, where $p = P(y_i = 1)$. Show that this log-odds equals $\mathbf{wx} + b$.

(b) Match each function on the left that plays a role in the model with its image on the right. Hint: The *image* of a function is the set of all output values it may produce.

i. _______	$f_1(\mathbf{x}) = \mathbf{wx} + b$ for $\mathbf{x} \in \mathbb{R}^D$	1. $[0, 1]$, the interval from 0 to 1
ii. _______	$f_2(t) = \frac{1}{1+e^{-t}}$ for $t \in \mathbb{R}$	2. $\mathbb{R}_+$, the positive real numbers
iii. _______	$f_3(t) = e^{-t}$ for $t \in \mathbb{R}$	3. $\mathbb{R}$, the real numbers

(c) I ran some Python/scikit-learn code to make the model pictured here:



i. Match each code line on the left, with its output on the right.

_______ `model.intercept_`   1. array([0, 0, 0, 1])

_______ `model.coef_[0]`   2. array([0.003, 0.5, 0.5, 0.997])

_______ `model.predict(X)`   3. array([5.832])

_______ `model.predict_proba(X)[:, 1]`   4. array([0.])

ii. How do we classify a new point at $x = -0.5$ if using a decision threshold of 0.7?

_______ $\hat{y} = 0$

_______ $\hat{y} \approx 0.05$

_______ $\hat{y} \approx 0.95$

_______ $\hat{y} = 1$

5. e.g. Consider using $\mathbf{w} = \left(X^T X\right)^{-1} X^T \mathbf{y}$ to find the line fitting the points $(0, -1)$ and $(2, 3)$.

Fill in these matrices to get started on using $\mathbf{w} = \left(X^T X\right)^{-1} X^T \mathbf{y}$ to find the line.

$$X = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}, \mathbf{y} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

(You should not continue the computation to find the line, which is $y = 2x - 1$.[1])

[1] What did one regression coefficient say to the other?
(This question was in a footnote in the notes, but I forgot to discuss it. You may answer if you wish, for 0 points.)

(a) ________ I'm partial to you.

(b) ________ We do not have a sense of humor we're aware of.

For another 0 points and only if you wish, write something here to make your graders smile.

6. Our hard-margin SVM used the constraints $\begin{cases} \mathbf{w}\mathbf{x}_i + b \geq 1 & \text{if } y_i = +1 \\ \mathbf{w}\mathbf{x}_i + b \leq -1 & \text{if } y_i = -1 \end{cases}$, for $i = 1, \ldots, N$.

Consider a new model, SVM_{new}, that uses the constraints $\begin{cases} \mathbf{w}\mathbf{x}_i + b \geq 0 & \text{if } y_i = +1 \\ \mathbf{w}\mathbf{x}_i + b < 0 & \text{if } y_i = -1 \end{cases}$.

Changing from the hard-margin SVM to SVM_{new} would do what to the margin?

_______ Increase it.

_______ Decrease it.

_______ Leave it unchanged.

_______ We cannot say without more information.

7. Consider a database consisting of these three examples:

name $\mathbf{x}$	age y
`Karolin`	20
`Kathrin`	30
`Kerstin`	40

We want to estimate `Kathryn`'s age from her name, supposing her name is a corrupted version of one of the names in the database. (It was corrupted, e.g., by a typographical error.)

(a) Find the Hamming distance between `Kathryn` and each of the other three names.

name	Hamming distance to `Kathryn`
`Karolin`	
`Kathrin`	
`Kerstin`	

(b) Use 2-NN (two nearest neighbors) regression to estimate `Kathryn`'s age from her name. `Kathryn`'s age is about _______.