(1 point for easily legible writing on this cover sheet.)

NetID: ______________________ (mine is `jgillett` from `jgillett@wisc.edu`)

Last name: ____________________________ First name: ______________________

Mark your lecture with “X”:

______ TuTh 9:30-10:45

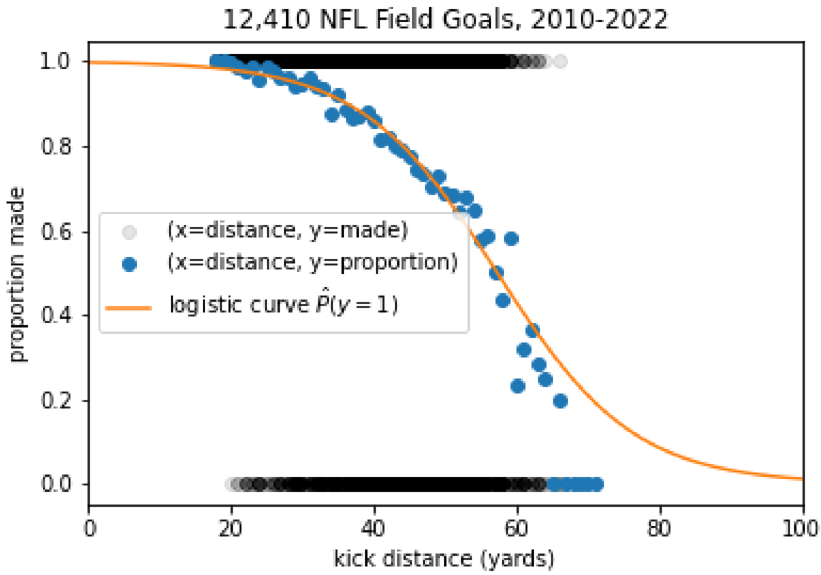
______ TuTh 11:00-12:15

STAT 451 Midterm Exam Instructions:

1. Please sit in alternating columns.
2. Do not open the exam until I say “go.”
3. Put away everything except a pencil or pen, a calculator, and your one-page (two sides) notes sheet.
4. Show your work. Correct answers without at least a minimal version of the work normally required may receive no credit.
5. If a question is ambiguous, resolve the ambiguity in writing. We will consider grading accordingly.
6. The exam ends when I call time. If you continue writing after I call time, you risk a penalty. (The alternative, that you get more time than your peers, is unfair.)
7. If a question is ambiguous, resolve the ambiguity in writing. We will consider grading accordingly. e.g.
   - I think “average” refers to the population mean μ (not the sample mean $\bar{X}$).
   - I think “linear regression” refers to OLS, not ridge or lasso.

Question	Points	Earned
Q0 (cover)	1	
Q1	4	
Q2	5	
Q3	15	
Q4	14	
Q5	10	
Q6	9	
Q7	20	
Q8	10	
Q9	12	
Total	100	

1. From the logistic regression model represented in the figure, estimate the likelihood of an NFL field goal kicker making three field goals in a row, one from 20 yards, one from 40, and one from 60. We may suppose these attempts are independent and make other reasonable simplifiying assumptions.



P(NFL kicker makes the three field goals) = _____________

2. Consider applying gradient descent with step size $\alpha = 0.5$ to find the $\mathbf{x}$ that minimizes the function $f(\mathbf{x}) = f\left(x^{(1)}, x^{(2)}, x^{(3)}\right) = \left(x^{(1)} - 1\right)^2 + \left(x^{(2)} + 2\right)^2 + \left(3 - x^{(3)}\right)^2$ starting from $\mathbf{x}_0 = (1, 1, 1)$. Find the value $\mathbf{x}_1$ after one iteration.

3. Here are some questions on support vector machines.

(a) Suppose we have a soft-margin SVM for which $\mathbf{w} = (2, 3)$ and $b = -1$. How does the SVM classify $(\mathbf{x} = (2, 1), y = 1)$?

(b) Suppose we have some training data $\{(\mathbf{x}_i, y_i)\}_{i=1}^N$ in matrices `X` and `y`. We have plotted the data with $y = -1$ examples red and $y = 1$ examples blue. Which line of code gives the best model for predicting new examples?

For each question, write the best answer from among these lines labeled “A” thorugh “H”. You may not use an answer more than once.

A: `clf = svm.SVC(kernel='linear', C=1); clf.fit(X, y)`

B: `clf = svm.SVC(kernel='linear', C=1000); clf.fit(X, y)`

C: `clf = svm.SVC(kernel="rbf", C=1, gamma=1); clf.fit(X, y)`

D: `clf = svm.SVC(kernel="rbf", C=1, gamma=10); clf.fit(X, y)`

E: `clf = svm.SVC(kernel="euclidean", C=1, gamma=2); clf.fit(X, y)`

F: `clf = svm.SVC(kernel="euclidean", C=1000, gamma=2); clf.fit(X, y)`

G: `clf = svm.SVC(kernel="gini", C=1); clf.fit(X, y)`

H: `clf = svm.SVC(kernel="gini", C=1000); clf.fit(X, y)`

i. _______ The data are two linearly-separable clouds of points, one red and one blue.

ii. _______ The red points are scattered between $x = -3$ and $x = 3$ and roughly along $y = x^2$, a parabola with vertex $(0, 0)$ that opens up. The blue points are scattered between $x = -3$ and $x = 3$ and roughly along $y = x^2 + 2$, a parabola with vertex $(0, 2)$ that opens up.

iii. _______ The data consist of two clouds of points, one red and one blue, that are linearly-separable except for a few outliers of each color.

iv. _______ The data consist of mixed red and blue points scattered randomly in the region $x \in [0, 1]$ and $y \in [0, 1]$.

4. Consider finding the linear regression line by hand for the points $(1, 3), (2, 5), (3, 4)$. Match each mathematical quantity on the left with its value on the right.

(a) $\mathbf{X} =$ _______

(b) $\mathbf{X}^T\mathbf{X} =$ _______

(c) $\mathbf{y} =$ _______

(d) $\mathbf{w} =$ _______

(e) $\hat{\mathbf{y}} =$ _______

(f) $f_{\mathbf{w},b}(3) =$ _______

(g) intercept $=$ _______

(A) 4.5

(B) $\begin{bmatrix} 3 & 6 \\ 6 & 14 \end{bmatrix}$

(C) $\begin{bmatrix} 3 \\ 0.5 \end{bmatrix}$

(D) $\begin{bmatrix} 3 \\ 5 \\ 4 \end{bmatrix}$

(E) $\begin{bmatrix} 3.5 \\ 4.0 \\ 4.5 \end{bmatrix}$

(F) 3

(G) $\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$

5. Consider 3-NN (three nearest neighbors) using the Minkowski distance with $p = 1$.

(a) Find the distance from $\mathbf{z} = (3, 4)$ to each of the other points $\mathbf{x}$:

$\mathbf{x}$	y	Distance from $\mathbf{z}$ to $\mathbf{x}$
$(-1, -1)$	1	
(0, 1)	0	
(1, 0)	0	
$(2, 3)$	1	

(b) How does 3-NN classifiy $\mathbf{z}$?

(c) What y value does 3-NN regression predict for $\mathbf{z}$?

(d) How does weighted 3-NN classifiy $\mathbf{z}$?

6. Suppose we have run this code, which reads the `data_string` into a data frame `df` and then creates `X` and `y` from `df`:

```
from io import StringIO
import pandas as pd

data_string = """
x1,     x2,  x3,  x4,  y
0,   -0.01,   1,  13,  5
1,   -0.00,   0,  12,  6
2,   -0.03,   3,  11,  7
3,   -0.02,   2,  10,  8
"""
df = pd.read_csv(StringIO(data_string), sep='\s*,\s+', engine='python')
X = df[['x1', 'x2', 'x3', 'x4']]
y = df.y
```

Hint: I answered these questions without doing calculations. I just inspected the data.

(a) The following code prints some subset of the data.

```
from sklearn.feature_selection import VarianceThreshold, SelectKBest
from sklearn.feature_selection import r_regression, f_regression

selector = VarianceThreshold(threshold=0.1)
selector.fit_transform(X)
```

Circle the names of the features that are displayed by the last line:

x1,   x2,   x3,   x4,   y

(b) The following code prints some subset of the data.

```
selector = SelectKBest(score_func=r_regression, k=2)
selector.fit_transform(X, y)
```

Circle the names of the features that are displayed by the last line:

x1,   x2,   x3,   x4,   y

(c) The following code prints some subset of the data.

```
selector = SelectKBest(score_func=f_regression, k=2)
selector.fit_transform(X, y)
```

Circle the names of the features that are displayed by the last line:

x1,   x2,   x3,   x4,   y

7. Mark each statement TRUE or FALSE by circling the appropriate choice.

(a) TRUE / FALSE In linear regression, a reasonable alternative to the cost function *mean squared error* $= \frac{1}{N}\sum_{i=1}^{N}[f_{\mathbf{w},b}(\mathbf{x}_i) - y_i]^2$ is *sum of squared error* $= \sum_{i=1}^{N}[f_{\mathbf{w},b}(\mathbf{x}_i) - y_i]^2$.

(b) TRUE / FALSE Using gradient descent to minimize $z = f(\mathbf{x})$ can be slow because the algorithm requires many calls to f, which is expensive if N (number of examples) is large or D (number of features) is large.

(c) TRUE / FALSE For the soft-margin SVM with decision boundary $\mathbf{wx} + b = 0$ where $\mathbf{w} = (1, 2)$ and $b = 3$, the example $(\mathbf{x}, y) = ((4, 5), -1)$ has hinge loss 18.

(d) TRUE / FALSE For training data $\{(\mathbf{x}, y)\}$ such that $\mathbf{x}_i \neq \mathbf{x}_j$ for all i and j, we can build a 3NN model that classifies the training examples without error.

(e) TRUE / FALSE If we train a hard-margin linear SVM on linearly separable data, then discard training examples which are support vectors, and then train a new SVM on the remaining examples, the first SVM will have a wider "road" than the second.

(f) TRUE / FALSE A linear SVM with decision boundary $(1, 2, 2) \cdot \mathbf{x} - 2 = 0$ has a smaller margin between $+1$ and -1 support vectors than one with boundary $(1, 4, 8) \cdot \mathbf{x} + 3 = 0$.

(g) TRUE / FALSE Every decision tree regression function is a step function.

Hint: A *step function* is a function that is constant over each of one or more intervals.

(h) TRUE / FALSE Every k-NN regression function is a step function.

Hint: A *step function* is a function that is constant over each of one or more intervals.

(i) TRUE / FALSE In logistic regression, we use the natural log function to facilitate finding a closed-form expression for the coefficients $\mathbf{w}$ and b in terms of the data.

(j) TRUE / FALSE Gradient descent can fail to converge to a global minimum if it gets stuck in a local minimum.

8. Here are some questions on feature engineering.

(a) Do min-max rescaling on feature `x`:

(input) `x`	(output) `x_rescaled`
1	
0	
3	

(b) Use one-hot encoding to transform the categorical feature `weather` into binary features with reasonable names.

(input) `weather`	(output)
cloudy	
rainy	
sunny	
cloudy	

9. Here are some questions about decision trees.

(a) Consider a classification decision tree node containing the set of examples $S = \{(\mathbf{x}, y)\}$ where $\mathbf{x} = (x_1, x_2)$:

S

x_1	x_2	x_3	y
2	11	12	0
1	8	14	1
0	6	17	1
3	10	15	0
4	7	16	1
5	9	13	0

i. The entropy of this node in bits is _______.

ii. The (feature, threshold) pair (j, t) that yields the best split for this node is feature $j =$ _______ and threshold $t =$ _______.

(b) Consider a regression decision tree with `max_depth=1` (that is, the root node is split once into two leaves) made from the set of examples $S = \{(\mathbf{x}, y)\}$ where $\mathbf{x} = x_1$:

S

x_1	y
0	1
1	2
2	11
3	12
4	13
5	14

What value does this tree predict for $x_1 = 2.5$? _____________