STATS426 Fall 2019: Topics and review problems for Exam 1

The topics and questions given here are meant as a guide to help you study for Exam 1, which will be on Thursday, October 17, 2019. The actual exam may include some problems that are more difficult (and some that are easier). See the chapter summaries and supplementary problems in the Rice textbook as well.

The exam will include a number of exercises that are similar to homework problems. You should review lecture notes, readings, and homework exercises and make sure you understand how to solve the problems from your homeworks.

How to use this guide:

a) Look through each of the suggested problems and make sure you understand what the problem is asking.

b) If you see a problem that seems confusing or difficult, try to do it first. Please note that solutions to the suggested problems (listed below) will NOT be posted; some of the problems have answers in the back of the text, which you can use to check your work.

c) Go through the Chapter Review section of Chapter 7 and Chapter 8 (although not all of either of these chapters will be covered; see my announcement from October 2 or see below for more information), and make sure you understand the definitions and key concepts. Note that the first 6 chapters are review from STATS425 and are fair game for this exam.

d) Review homework problems carefully.

e) Focus on how particular important results are derived and make sure you understand the underlying process, rather than simply memorizing formulas.

f) Make up your own cheat sheet of key facts, formula, etc. You will not be allowed to use this cheat sheet in the exam, but the mere process of making it is one of the best things you can do to organize your thoughts.

TOPICS AND SUGGESTED EXERCISES

**Topic/Chapter**: Review of STATS425: basic probability, conditional probability and conditional expectation, joint distributions, independence, multivariate normality, order statistics, Chebyshev’s inequality, etc.

Review Homeworks 1 and 2 for examples and look through Chapters 1–6.

**Topic/Chapter**: Rice, Chapter 7, Sections 7.1–7.3: Survey sampling

(a) Population vs. sample

(b) Population parameters: mean, variance, interpretation in both the Bernoulli and general case

(c) Simple random sampling, with and without replacement
(d) Estimators for various population parameters (such as the sample mean, sample variance for the population mean and variance, etc.)

(e) Unbiased estimators and the mean square error of an estimator

(f) Distribution and properties of the sample mean $\bar{X}$, with and without replacement

(g) The mean and variance of $\bar{X}$ and how to compute it, with and without replacement

(h) Estimation of the population variance: mean squared deviation $\hat{\sigma}^2$ and sample variance $s^2$, and their properties, including the expected value of $s^2$ and $\hat{\sigma}^2$

(i) Approximate distribution of $\bar{X}$, under reasonable assumptions on sample size $n$ and population size $N$

(j) Using the approximate distribution of the sample mean to determine confidence intervals for the population mean

(k) Approaches to estimating the standard error of an estimator

(l) Bivariate populations—basic set-up of pairs $(x, y)$ of measurements on a collection of $N$ objects

(m) Population parameters and their sample estimates for this bivariate data—population covariance $\sigma_{xy}$ and sample covariance

**Suggested problems:** (in addition to the assigned homework problems) Rice, Chapter 7, Exercises 11, 12, 21, 23, 24, 35, 36, 37.

**Topic/Chapter:** Rice, Chapter 8, Sections 8.1–8.5: Parametric estimation

(a) Basic idea of parametric estimation based on i.i.d data $X_1, \cdots, X_n$ drawn from a distribution $F_\theta$, where $\theta$ is some unknown parameter; notion of consistency for an estimator

(b) Method-of-moments estimation: computing $k$-th population moments in terms of the parameters and then solving for the parameters in terms of the moments; using sample moments to approximate population moments. Examples: Poisson, gamma, normal, Bernoulli, etc.

(c) Maximum likelihood estimation: likelihood function, log-likelihood; maximizing the likelihood function as a function of the parameter and obtaining the maximum likelihood estimate $\hat{\theta}$. Examples: Poisson, gamma, normal, binomial, uniform, Bernoulli, etc.

(d) Large-sample properties of the MLE, including asymptotic normality, consistency, and the relationship between the asymptotic variance of the MLE and the Fisher information

**Suggested problems:** (in addition to the assigned homework problems) Rice, Chapter 8, 6(a), 18 (a)–(b), 19 (a)–(b), 30, 32.