STAT606
Computing for Data Science and Statistics

Lecture 2: Conditionals, Recursion and Iteration

Boolean Expressions

Boolean expressions evaluate the truth/falsity of a statement

Python supplies a special Boolean type, `bool`

variable of type `bool` can be either `True` or `False`

```
1 type(True)
```

```
bool
```

```
1 type(False)
```

```
bool
```

"Boolean" refers to George Boole, an English logician and philosopher.
https://en.wikipedia.org/wiki/George_Boole

Boolean Expressions

Comparison operators available in Python:

```
1 x == y # x is equal to y
2 x != y # x is not equal to y
3 x > y # x is strictly greater than y
4 x < y # x is strictly less than y
5 x >= y # x is greater than or equal to y
6 x <= y # x is less than or equal to y
```

Expressions involving comparison operators evaluate to a Boolean.

```
1 x = 10
2 y = 20
3 x == y
```

```
False
```

```
1 x != y
```

```
True
```

```
1 x < x
```

```
False
```

```
1 x <= x
```

```
True
```

Note: In true Pythonic style, one can compare many types, not just numbers. Most obviously, strings can be compared, with ordering given alphabetically.

Boolean Expressions

Can combine Boolean expressions into larger expressions via **logical operators**

In Python: `and`, `or` and `not`

```
1 x = 10
2 x < 20 and x > 0
```

```
True
```

```
1 x > 100 and x > 0
```

```
False
```

```
1 x > 100 or x > 0
```

```
True
```

```
1 not x > 0
```

```
False
```

Boolean Expressions

Can combine Boolean expressions into larger expressions via **logical operators**

In Python: `and`, `or` and `not`

```
1 x = 10
2 x < 20 and x > 0
```

```
True
```

```
1 x > 100 and x > 0
```

```
False
```

```
1 x > 100 or x > 0
```

```
True
```

```
1 not x > 0
```

```
False
```

Note: technically, any nonzero number or any nonempty string will evaluate to `True`, but you should avoid using logical operators on anything that isn't Boolean.

```
1 1 and x > 0
```

```
True
```

```
1 0 and x > 0
```

```
0
```

```
1 'cat' and x > 0
```

```
True
```

```
1 '' and x > 0
```

```
''
```

Boolean Expressions: Example

Let's see Boolean expressions in action

```
1  def is_even(n):
2      # Returns a boolean.
3      # Returns True if and only if
4      # n is an even number.
5      return n % 2 == 0
```

Reminder: `x % y` returns the remainder when `x` is divided by `y`.

Note: in practice, we would want to include some extra code to check that `n` is actually a number, and to "fail gracefully" if it isn't, e.g., by throwing an error with a useful error message. More about this in future lectures.

```
1  is_even(0)
```

```
True
```

```
1  is_even(1)
```

```
False
```

```
1  is_even(8675309)
```

```
False
```

```
1  is_even(-3)
```

```
False
```

```
1  is_even(12)
```

```
True
```

Conditional Expressions

Sometimes we want to do different things depending on certain conditions

```
1 x = 10
2 if x > 0:
3     print 'x is bigger than 0'
4 if x > 1:
5     print 'x is bigger than 1'
6 if x > 100:
7     print 'x is bigger than 100'
8 if x < 100:
9     print 'x is less than 100'
```

```
x is bigger than 0
x is bigger than 1
x is less than 100
```

Note: Oops! I used Python 2 code in this slide! Notice that `print` is being used as a statement, not a function. To run this in Python 3, you'll need to change this code to read `print('x is bigger than 0')`, etc.

Conditional Expressions

Sometimes we want to do different things depending on certain conditions

```
1 x = 10
2 if x > 0:
3     print 'x is bigger than 0'
4 if x > 1:
5     print 'x is bigger than 1'
6 if x > 100:
7     print 'x is bigger than 100'
8 if x < 100:
9     print 'x is less than 100'
```

This is an **if-statement**.

```
x is bigger than 0
x is bigger than 1
x is less than 100
```

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Conditional Expressions

Sometimes we want to do different things depending on certain conditions

```
1 x = 10
2 if x > 0:
3     print 'x is bigger than 0'
4 if x > 1:
5     print 'x is bigger than 1'
6 if x > 100:
7     print 'x is bigger than 100'
8 if x < 100:
9     print 'x is less than 100'
```

```
x is bigger than 0
x is bigger than 1
x is less than 100
```

This Boolean expression is called the **test condition**, or just the **condition**.

Note: Oops! I used Python 2 code in this slide! Notice that `print` is being used as a statement, not a function. To run this in Python 3, you'll need to change this code to read `print('x is bigger than 0')`, etc.

Conditional Expressions

Sometimes we want to do different things depending on certain conditions

```
1 x = 10
2 if x > 0:
3     print 'x is bigger than 0'
4 if x > 1:
5     print 'x is bigger than 1'
6 if x > 100:
7     print 'x is bigger than 100'
8 if x < 100:
9     print 'x is less than 100'
```

```
x is bigger than 0
x is bigger than 1
x is less than 100
```

If the condition evaluates to `True`, then Python runs the code in the body of the if-statement.

Note: Oops! I used Python 2 code in this slide! Notice that `print` is being used as a statement, not a function. To run this in Python 3, you'll need to change this code to read `print('x is bigger than 0')`, etc.

Conditional Expressions

Sometimes we want to do different things depending on certain conditions

```
1 x = 10
2 if x > 0:
3     print 'x is bigger than 0'
4 if x > 1:
5     print 'x is bigger than 1'
6 if x > 100:
7     print 'x is bigger than 100'
8 if x < 100:
9     print 'x is less than 100'
```

```
x is bigger than 0
x is bigger than 1
x is less than 100
```

If the condition evaluates to `False`, then Python skips the body and continues running code starting at the end of the if-statement.

Note: Oops! I used Python 2 code in this slide! Notice that `print` is being used as a statement, not a function. To run this in Python 3, you'll need to change this code to read `print('x is bigger than 0')`, etc.

Conditional Expressions

Sometimes we want to do different things depending on certain conditions

```
1 x = 10
2 if x > 0:
3     print 'x is bigger than 0'
4 if x > 1:
5     print 'x is bigger than 1'
6 if x > 100:
7     print 'x is bigger than 100'
8 if x < 100:
9     print 'x is less than 100'
```

```
x is bigger than 0
x is bigger than 1
x is less than 100
```

Note: the body of a conditional statement can have any number of lines in it, but it must have at least one line. To do nothing, use the `pass` keyword.

```
1 y = 20
2 if y > 0:
3     pass # TODO: handle positive numbers!
4 if y < 100:
5     print 'y is less than 100'
```

```
y is less than 100
```

Conditional Expressions

More complicated logic can be handled with **chained conditionals**

```
1 def pos_neg_or_zero(x):
2     if x < 0:
3         print 'That is negative'
4     elif x == 0:
5         print 'That is zero.'
6     else:
7         print 'That is positive'
8 pos_neg_or_zero(1)
```

```
That is positive
```

```
1 pos_neg_or_zero(0)
2 pos_neg_or_zero(-100)
3 pos_neg_or_zero(20)
```

```
That is zero.
That is negative
That is positive
```

Note: Oops! Still Python 2! You need to change the print statements to functions to get this to run in Python 3.

Conditional Expressions

More complicated logic can be handled with **chained conditionals**

```
1 def pos_neg_or_zero(x):
2     if x < 0:
3         print 'That is negative'
4     elif x == 0:
5         print 'That is zero.'
6     else:
7         print 'That is positive'
8 pos_neg_or_zero(1)
```

```
That is positive
```

This is treated as a single if-statement.

```
1 pos_neg_or_zero(0)
2 pos_neg_or_zero(-100)
3 pos_neg_or_zero(20)
```

```
That is zero.
That is negative
That is positive
```

Note: Oops! Still Python 2! You need to change the print statements to functions to get this to run in Python 3.

Conditional Expressions

More complicated logic can be handled with **chained conditionals**

```
1 def pos_neg_or_zero(x):
2     if x < 0:
3         print 'That is negative'
4     elif x == 0:
5         print 'That is zero.'
6     else:
7         print 'That is positive'
8 pos_neg_or_zero(1)
```

```
That is positive
```

If this expression evaluates to `True`...

```
1 pos_neg_or_zero(0)
2 pos_neg_or_zero(-100)
3 pos_neg_or_zero(20)
```

```
That is zero.
That is negative
That is positive
```

Note: Oops! Still Python 2! You need to change the print statements to functions to get this to run in Python 3.

Conditional Expressions

More complicated logic can be handled with **chained conditionals**

```
1 def pos_neg_or_zero(x):
2     if x < 0:
3         print 'That is negative'
4     elif x == 0:
5         print 'That is zero.'
6     else:
7         print 'That is positive'
8 pos_neg_or_zero(1)
```

```
That is positive
```

...then this block of code is executed...

```
1 pos_neg_or_zero(0)
2 pos_neg_or_zero(-100)
3 pos_neg_or_zero(20)
```

```
That is zero.
That is negative
That is positive
```

Note: Oops! Still Python 2! You need to change the print statements to functions to get this to run in Python 3.

Conditional Expressions

More complicated logic can be handled with **chained conditionals**

```
1 def pos_neg_or_zero(x):
2     if x < 0:
3         print 'That is negative'
4     elif x == 0:
5         print 'That is zero.'
6     else:
7         print 'That is positive'
8 pos_neg_or_zero(1)
```

```
That is positive
```

...and then Python exits the if-statement

```
1 pos_neg_or_zero(0)
2 pos_neg_or_zero(-100)
3 pos_neg_or_zero(20)
```

```
That is zero.
That is negative
That is positive
```

Note: Oops! Still Python 2! You need to change the print statements to functions to get this to run in Python 3.

Conditional Expressions

More complicated logic can be handled with **chained conditionals**

```
1 def pos_neg_or_zero(x):
2     if x < 0:
3         print 'That is negative'
4     elif x == 0:
5         print 'That is zero.'
6     else:
7         print 'That is positive'
8 pos_neg_or_zero(1)
```

```
That is positive
```

If this expression evaluates to `False`...

```
1 pos_neg_or_zero(0)
2 pos_neg_or_zero(-100)
3 pos_neg_or_zero(20)
```

```
That is zero.
That is negative
That is positive
```

Note: Oops! Still Python 2! You need to change the print statements to functions to get this to run in Python 3.

Conditional Expressions

More complicated logic can be handled with **chained conditionals**

```
1 def pos_neg_or_zero(x):
2     if x < 0:
3         print 'That is negative'
4     elif x == 0:
5         print 'That is zero.'
6     else:
7         print 'That is positive'
8 pos_neg_or_zero(1)
```

```
That is positive
```

```
1 pos_neg_or_zero(0)
2 pos_neg_or_zero(-100)
3 pos_neg_or_zero(20)
```

```
That is zero.
That is negative
That is positive
```

Note: `elif` is short for **else if**.

...then we go to the condition. If this condition fails, we go to the next condition, etc.

Note: Oops! Still Python 2! You need to change the print statements to functions to get this to run in Python 3.

Conditional Expressions

More complicated logic can be handled with **chained conditionals**

```
1 def pos_neg_or_zero(x):
2     if x < 0:
3         print 'That is negative'
4     elif x == 0:
5         print 'That is zero.'
6     else:
7         print 'That is positive'
8 pos_neg_or_zero(1)
```

```
That is positive
```

If all the other tests fail, we execute the block in the `else` part of the statement.

```
1 pos_neg_or_zero(0)
2 pos_neg_or_zero(-100)
3 pos_neg_or_zero(20)
```

```
That is zero.
That is negative
That is positive
```

Note: Oops! Still Python 2! You need to change the print statements to functions to get this to run in Python 3.

Conditional Expressions

Conditionals can also be nested

```
if x==y:
    print 'x is equal to y'
else:
    if x > y:
        print 'x is greater than y'
    else:
        print 'y is greater than x'
```

This if-statement...

Note: Oops! Still Python 2! You need to change the print statements to functions to get this to run in Python 3.

Conditional Expressions

Conditionals can also be nested

```
if x==y:
    print 'x is equal to y'
else:
    if x > y:
        print 'x is greater than y'
    else:
        print 'y is greater than x'
```

This if-statement...

...contains another if-statement.

Note: Oops! Still Python 2! You need to change the print statements to functions to get this to run in Python 3.

Conditional Expressions

Often, a nested conditional can be simplified

When this is possible, I recommend it for the sake of your sanity, because debugging complicated nested conditionals is tricky!

These two if-statements are equivalent, in that they do the same thing!

```
1 if x > 0:
2     if x < 10:
3         print 'x is a positive single-digit number.'
```

But the second one is (arguably) preferable, as it is simpler to read.

```
1 if 0 < x and x < 10:
2     print 'x is a positive single-digit number.'
```

Recursion

A function is allowed to call itself, in what is termed **recursion**

```
1 def countdown(n):
2     if n <= 0:
3         print'We have lift off!'
4     else:
5         print n
6         countdown(n-1)
```

Countdown calls itself!

```
1 countdown(10)
```

```
10
9
8
7
6
5
4
3
2
1
We have lift off!
```

But the key is that each time it calls itself, it is passing an argument with its value decreased by 1, so eventually, `n <= 0` is true.

With a small change, we can make it so that `countdown(1)` encounters an **infinite recursion**, in which it repeatedly calls itself.

```
1 def countdown(n):
2     if n <= 0:
3         print'We have lift off!'
4     else:
5         print n
6         countdown(n)
```

```
1 countdown(10)
```

```
---------------------------------------------------------------------------
RuntimeError                              Traceback (most recent call last)
<ipython-input-163-a972007fb272> in <module>()
----> 1 countdown(10)

<ipython-input-162-33965ef63097> in countdown(n)
      4     else:
      5         print n
----> 6         countdown(n)

... last 1 frames repeated, from the frame below ...

<ipython-input-162-33965ef63097> in countdown(n)
      4     else:
      5         print n
----> 6         countdown(n)

RuntimeError: maximum recursion depth exceeded
```

Repeated actions: Iteration

Recursion is the first tool we've seen for performing repeated operations

But there are better tools for the job: `while` and `for` loops.

```
1 def countdown(n):
2     while n>0:
3         print n
4         n = n-1
5     print 'We have lift off!'
```

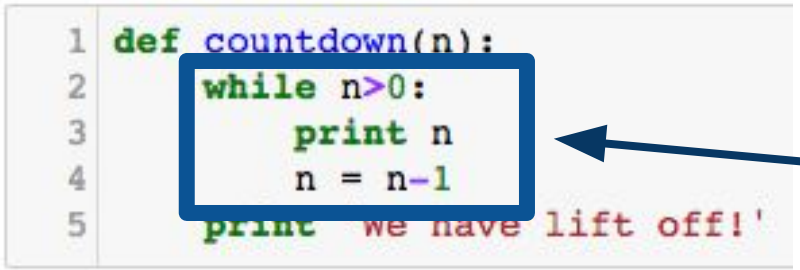
```
1 countdown(10)
```

```
10
9
8
7
6
5
4
3
2
1
We have lift off!
```

Repeated actions: Iteration

Recursion is the first tool we've seen for performing repeated operations

But there are better tools for the job: `while` and `for` loops.



This block specifies a while-loop. So long as the condition is true, Python will run the code in the body of the loop, checking the condition again at the end of each time through.

Repeated actions: Iteration

Recursion is the first tool we've seen for performing repeated operations

But there are better tools for the job: `while` and `for` loops.

```
1 def countdown(n):
2     while n>0:
3         print n
4         n = n-1
5     print 'We have lift off!'
```

```
1 countdown(10)
```

```
10
9
8
7
6
5
4
3
2
1
We have lift off!
```

Repeated actions: Iteration

Recursion is the first tool we've seen for performing repeated operations

But there are better tools for the job: `while` and `for` loops.

```
1 def countdown(n):
2     while n>0:
3         print n
4         n = n-1
5     print 'We have lift off!'
```

Warning: Once again, there is a danger of creating an **infinite loop**. If, for example, `n` never gets updated, then when we call `countdown(10)` , the condition `n>0` will always evaluate to `True`, and we will never exit the while-loop.

```
1 countdown(10)
```

```
10
9
8
7
6
5
4
3
2
1
We have lift off!
```

Repeated actions: Iteration

```
1 def collatz(n):
2     while n!=1:
3         print(n)
4         if n % 2 == 0:
5             n = n/2
6         else:
7             n = 3*n+1
```

```
1 collatz(20)
```

```
20
10
5
16
8
4
2
```

One always wants to try and ensure that a while loop will (eventually) terminate, but it's not always so easy to know!
https://en.wikipedia.org/wiki/Collatz_conjecture

"Mathematics may not be ready for such problems."
Paul Erdős

Repeated actions: Iteration

We can also terminate a while-loop using the `break` keyword

```
1 a = 4
2 x = 3.5
3 epsilon = 10**-6
4 while True:
5     print(x)
6     y = (x + a/x)/2
7     if abs(x-y) < epsilon:
8         break
9     x=y # update to our new estimate
```

```
3.5
2.32142857143
2.02225274725
2.00012243394
2.00000000375
```

The `break` keyword terminates the current loop when it is called.

Newton-Raphson method:
https://en.wikipedia.org/wiki/Newton's_method

Repeated actions: Iteration

We can also terminate a while-loop using the `break` keyword

```
1  a = 4
2  x = 3.5
3  epsilon = 10**-6
4  while True:
5      print(x)
6      x = (x + a/x)/2
7      if abs(x-y) < epsilon:
8          break
9      y = x # update to our new estimate
```

```
3.5
2.32142857143
2.02225274725
2.00012243394
2.00000000375
```

Notice that we're not testing for equality here. That's because testing for equality between pairs of floats is dangerous. When I write `x=1/3`, for example, the value of `x` is actually only an approximation to the number 1/3.

Newton-Raphson method:
https://en.wikipedia.org/wiki/Newton's_method

Aside: floating-point arithmetic

The number 1/3 has no finite decimal representation, i.e., 1/3 = 0.33333333…

...but our computer uses only finitely many bits to represent a float…

One solution: symbolic computation https://en.wikipedia.org/wiki/Computer_algebra

More practical: **floating-point arithmetic**

https://en.wikipedia.org/wiki/Floating-point_arithmetic

Idea: essentially, pick out a finite set of decimal numbers...

...round variables, solutions, etc to the nearest such number

Drawback: introduces rounding errors e.g., 0.333333333… becomes 0.333332.

For more, see Goldberg (1991), *What Every Computer Scientist Should Know About Floating-Point Arithmetic*, ACM Computing Surveys, **23**(1).