Lecture 11: *numpy* and *scipy*
Some examples adapted from A. Tewari @ UMichigan
Numerical computing in Python: numpy

One of a few increasingly-popular, free competitors to MATLAB


For MATLAB fans:
  https://docs.scipy.org/doc/numpy-dev/user/numpy-for-matlab-users.html

Closely related package scipy is for optimization
  See https://docs.scipy.org/doc/
Installing packages

So far, we have only used built-in modules
But there are many modules/packages that do not come preinstalled

Ways to install packages:

At the **conda prompt or in terminal**: conda install numpy
[https://conda.io/docs/user-guide/tasks/manage-pkgs.html](https://conda.io/docs/user-guide/tasks/manage-pkgs.html)

Using **pip (recommended)**: pip install numpy

Using UNIX/Linux package manager (not recommended)
From source (not recommended)
Installing packages with `pip`

```
keith@Steinhaus:~$ pip3 install beautifulsoup4
Collecting beautifulsoup4
  Downloading beautifulsoup4-4.6.0-py3-none-any.whl (86kB)
    100% |████████████████████████████████| 92kB 1.4MB/s
Installing collected packages: beautifulsoup4
Successfully installed beautifulsoup4-4.6.0
```

The above command installs the package `beautifulsoup4`. We will use that later in the semester. To install `numpy`, type the same command, but use `numpy` in place of `beautifulsoup4`.
**numpy data types**

Five basic numerical data types:
- boolean (`bool`)
- integer (`int`)
- unsigned integer (`uint`)
- floating point (`float`)
- complex (`complex`)

Many more complicated data types are available
  e.g., each of the numerical types can vary in how many bits it uses

https://docs.scipy.org/doc/numpy/user/basics.types.html
numpy data types

32-bit and 64-bit representations are distinct!

Data type followed by underscore uses the default number of bits. This default varies by system.

As a rule, it's best never to check for equality of floats. Instead, check whether they are within some error tolerance of one another.
numpy.array: numpy's version of Python array (i.e., list)

Can be created from a Python list...

```
1 np.array([1, 2, 3], dtype='uint')
array([1, 2, 3], dtype=uint64)
```

...by “shaping” an array...

```
1 np.zeros((2,3))
array([[ 0.,  0.,  0.],
       [ 0.,  0.,  0.]])
```

...by “ranges”...

```
1 np.arange(2, 3, 0.1, dtype='float')
array([ 2. ,  2.1,  2.2,  2.3,  2.4,  2.5,  2.6,  2.7,  2.8,  2.9])
```

...or reading directly from a file

see https://docs.scipy.org/doc/numpy/user/basics.creation.html
**numpy allows arrays of arbitrary dimension (tensors)**

1-dimensional arrays:

```python
1 x = np.arange(12)  # x=[1,2,...,12]
2 x
```

```
array([[ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9, 10, 11])
```

2-dimensional arrays (matrices):

```python
1 x.shape = (3,4)  # now x is a 3-by-4 matrix
2 x  # observe that shape fills the new matrix by row.
```

```
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

3-dimensional arrays (“3-tensor”):

```python
1 x.shape = (2,3,2)  # now x is a 2-by-3-by-2 "cube" of numbers
2 x
```

```
array([[[ 0,  1],
        [ 2,  3],
        [ 4,  5]],
       [[ 6,  7],
        [ 8,  9],
        [10, 11]]])
```
\texttt{numpy} allows arrays of arbitrary dimension (tensors)

1-dimensional arrays:

\begin{verbatim}
1 x = np.arange(12)  # x=[1,2,...,12]
2 x
\end{verbatim}

array([[ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9, 10, 11]])

2-dimensional arrays (matrices):

\begin{verbatim}
1 x.shape = (3,4)  # now x is a 3-by-4 matrix
2 x  # observe that shape fills the new matrix by row.
\end{verbatim}

array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])

3-dimensional arrays ("3-tensor"):  

\begin{verbatim}
1 x.shape = (2,3,2)  
2 x  # now x is a 2-by-3-by-2 "cube" of numbers
\end{verbatim}

array([[[ 0,  1],
       [ 2,  3],
       [ 4,  5]],
       [[ 6,  7],
       [ 8,  9],
       [10, 11]]])

Every numpy array has a \texttt{shape} attribute specifying its dimensions. For example, an array with shape (3,4) has three rows and four columns. An array with shape (2,3,2) is a 2-by-3-by-2 “box” of numbers.

Think of the shape of an array as specifying how many indices we need to pick out an entry of the array. For example, to pick out a number from a 3-by-4 matrix, we must specify a row and a column.
More on `numpy.arange` creation

`np.arange(x)`: array version of Python’s `range(x)`, like `[0, 1, 2, ..., x-1]`

`np.arange(x, y)`: array version of `range(x, y)`, like `[x, x+1, ..., y-1]`

`np.arange(x, y, z)`: array of elements `[x, y)` in `z`-size increments.

```
1 np.arange(10)
array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])

1 np.arange(5,10)
array([5, 6, 7, 8, 9])

1 np.arange(0,1,0.1)
array([ 0. ,  0.1,  0.2,  0.3,  0.4,  0.5,  0.6,  0.7,  0.8,  0.9])
```
More on `numpy.arange` creation

`np.arange(x)`: array version of Python’s `range(x)`, like `[0, 1, 2, ..., x-1]`

`np.arange(x, y)`: array version of `range(x, y)`, like `[x, x+1, ..., y-1]`

`np.arange(x, y, z)`: array of elements `[x, y)` in z-size increments.

Related useful functions, that give better/clearer control of start/endpoints and allow for multidimensional arrays:

https://docs.scipy.org/doc/numpy/reference/generated/numpy.linspace.html
https://docs.scipy.org/doc/numpy/reference/generated/numpy.ogrid.html
numpy array indexing is highly expressive

Not very relevant to us right now...

...but this will come up again in a few weeks when we cover TensorFlow.
More array indexing

```
x = np.reshape(np.arange(1,13), (3,4))
x
```

```
array([[1, 2, 3, 4],
       [5, 6, 7, 8],
       [9, 10, 11, 12]])
x[1]
```

```
array([5, 6, 7, 8])
x[:,(1,3)]
```

```
array([[2, 4],
       [6, 8],
       [10, 12]])
x[(0,2),(1,3)]
```

```
array([2, 12])
```

If we specify fewer than the number of indices, `numpy` assumes we mean `:` in the remaining indices.

**Warning:** if you’re used to MATLAB or R, this behavior will seem weird to you.

**From the documentation:** When the index consists of as many integer arrays as the array being indexed has dimensions, the indexing is straight forward, but different from slicing. Advanced indexes always are broadcast and iterated as one.  
https://docs.scipy.org/doc/numpy/reference/arrays.indexing.html#integer-array-indexing
More array indexing

Numpy allows MATLAB/R-like indexing by Booleans

```
1 x = np.arange(10)
2 x[x>7]
array([8, 9])
```

```
1 x[(x>7) or (x<2)]
```

ValueError Traceback (most recent call last)
<ipython-input-373-6b519499a034> in <module>()
----> 1 x[(x>7) or (x<2)]

ValueError: The truth value of an array with more than one element is ambiguous. Use a.any() or a.all()

Believe it or not, this error is by design! The designers of \texttt{numpy} were concerned about ambiguities in Boolean vector operations. In essence, should \( (x>7) \) or \( (x<2) \) be a vector of Booleans or a single Boolean?
Boolean operations: \texttt{np.any()}, \texttt{np.all()}

```python
def the
```
Boolean operations: `np.logical_and()`

`numpy` also has built-in Boolean vector operations, which are simpler/clearer at the cost of the expressiveness of `np.any()`, `np.all()`.

```python
1  x = np.arange(10)
2  x[np.logical_and(x>3,x<7)]
```

```
array([[4, 5, 6]])
```

```python
1  np.logical_or(x<3,x>7)
```

```
array([[ True,  True,  True, False, False, False, False, False, False,  True,  True]], dtype=bool)
```

```python
1  x[np.logical_xor(x>3,x<7)]
```

```
array([[0, 1, 2, 3, 7, 8, 9]])
```

```python
1  x[np.logical_not(x>3)]
```

```
array([[0, 1, 2, 3]])
```

This is an example of a numpy “universal function” (ufunc), which we’ll discuss more in a few slides.
Random numbers in numpy

np.random contains methods for generating random numbers

```python
np.random.random((2,3))
```

```
array([[ 0.61420793, 0.46363275, 0.22880783],
       [ 0.24268979, 0.13462754, 0.6026283 ]])
```

```python
np.random.normal(0,1,20)
```

```
array([ 1.31323138, 0.76807767, 1.92180038, -0.34121468, 0.72572401,
       1.0273551, -0.78435871, 0.42732636, 1.05947171, 0.23042635,
       0.3951938, 0.3595342, 0.14710555, 0.42279814, 0.84381846,
       1.06495165, -1.51074354, -0.16419861, 2.89275956, -1.18501386])
```

```python
np.random.uniform(0,1,(2,4))
```

```
array([[ 0.08399452, 0.03934797, 0.3603464 , 0.66361677],
       [ 0.33499095, 0.29427732, 0.14963153, 0.87892145]])
```

Lots more distributions:

https://docs.scipy.org/doc/numpy/reference/routines.random.html#distributions
np.random.choice(): random samples from data

np.random.choice(x, [size, replace, p])
Generates a sample of size elements from the array x, drawn with
(replace=True) or without (replace=False) replacement, with element
probabilities given by vector p.

```python
x = np.arange(1, 11)
for i in range(5):
    print(np.random.choice(x, 5, False, x/float(sum(x))))
```

```
[ 1  5 10  7  6]
[ 8  5  9  2  6]
[ 9  6  3  8 10]
[ 7  9 10  5  6]
[ 8  5  6  9  1]
```
**shuffle() vs permutation()**

`np.random.shuffle(x)`
- randomly permutes entries of `x` in place
- so `x` itself is changed by this operation!

`np.random.permutation(x)`
- returns a random permutation of `x`
- and `x` remains unchanged.

Compare with the Python `list.sort()` and `sorted()` functions.
Statistics in `numpy`

`numpy` implements all the standard statistics functions you’ve come to expect

```python
x = np.random.normal(0,1,100)
np.mean(x), np.median(x), np.std(x)

(-0.062724875643358866, -0.05261873350441526, 1.0556291754262765)

np.min(x), np.max(x), np.ptp(x)  # ptp gets max-min

(-3.1029568746428113, 1.9628924810049164, 5.0658493556477282)

np.std(x), np.var(x)

(1.0556291754262765, 1.1143529560111607)
```
Statistics in numpy

Numpy deals with NaNs more gracefully than MATLAB/R:

```python
1  x[5] = np.nan
2  np.mean(x)

nan
```

```python
1  np.nanmin(x), np.nanmax(x), np.nanstd(x), np.nanvar(x)
```

```text
(-3.1029568746428113,  
  1.9628924810049164,  
  1.0439479158102707,  
  1.0898272509246081)
```

NaN is short for “not a number”. NaNs typically arise either because of improper mathematical operations (e.g., dividing by zero) or to represent missing data.

For more statistical functions, see:
https://docs.scipy.org/doc/numpy-1.8.1/reference/routines.statistics.html
Probability and statistics in \texttt{scipy}

(Among) all the distributions you could possibly ever want:

- [Continuous distributions](https://docs.scipy.org/doc/scipy/reference/stats.html#continuous-distributions)
- [Multivariate distributions](https://docs.scipy.org/doc/scipy/reference/stats.html#multivariate-distributions)
- [Discrete distributions](https://docs.scipy.org/doc/scipy/reference/stats.html#discrete-distributions)

More statistical functions (moments, kurtosis, statistical tests):

- [Statistical functions](https://docs.scipy.org/doc/scipy/reference/stats.html#statistical-functions)

```python
import scipy.stats
x = np.random.normal(0, 1, 20)
scipy.stats.kstest(x, 'norm')
```

KstestResult(statistic=0.23182037538316391, pvalue=0.19897055187485568)

\texttt{scipy} is a distinct Python package, part of the \texttt{numpy} ecosystem.

Kolmogorov-Smirnov test
Matrix-vector operations in **numpy**

```
A = np.reshape(np.arange(1,13), (3,4))
x = np.ones(4)
A*x
```

```
array([[ 1.,  2.,  3.,  4.],
       [ 5.,  6.,  7.,  8.],
       [ 9., 10., 11., 12.]])
```

```
y = np.ones(3)
A*y
```

```
ValueError
```

```
Traceback (most recent call last)
<ipython-input-83-86c92ad89b88> in <module>()
     1 y = np.ones(3)
----> 2 A*y

ValueError: operands could not be broadcast together with shapes (3,4) (3,)
```

```
np.reshape(y, (3,1))*A
```

```
array([[ 1.,  2.,  3.,  4.],
       [ 5.,  6.,  7.,  8.],
       [ 9., 10., 11., 12.]])
```

Trying to multiply two arrays, and you get **broadcast** behavior, **not** a matrix-vector product.

Broadcast multiplication still requires that dimensions agree and all that.
Matrix-vector operations in `numpy`:

Create a numpy matrix from a numpy array. We can also create matrices from strings with MATLAB-like syntax. See documentation:

```python
A = np.matrix(np.reshape(np.arange(1,13),(3,4)))

matrix([[ 1,  2,  3,  4],
         [ 5,  6,  7,  8],
         [ 9, 10, 11, 12]])
```

Now matrix-vector and vector-matrix multiplication work as we want.

```python
x = np.ones((4,1))
A*x

matrix([[10.],
         [26.],
         [42.]])
```

Numpy matrices support a whole bunch of useful methods. See documentation:

numpy/scipy universal functions (ufuncs)

From the documentation:

A universal function (or ufunc for short) is a function that operates on ndarrays in an element-by-element fashion, supporting array broadcasting, type casting, and several other standard features. That is, a ufunc is a “vectorized” wrapper for a function that takes a fixed number of scalar inputs and produces a fixed number of scalar outputs.

https://docs.scipy.org/doc/numpy/reference/ufuncs.html

So ufuncs are vectorized operations, just like in R and MATLAB
ufuncs in action

List comprehensions are great, but they’re not well-suited to numerical computing.

```python
1 x = range(10)
2 x**2
```

```
Traceback (most recent call last)
  1 x = range(10)
----> 2 x**2

TypeError: unsupported operand type(s) for ** or pow(): 'list' and 'int'
```

Unlike Python lists, numpy arrays support vectorized operations.

```python
1 [x**2 for x in np.arange(10)]
[0, 1, 4, 9, 16, 25, 36, 49, 64, 81]
```

```python
1 x = np.arange(10)
2 x**2
```

```
array([[ 0,  1,  4,  9, 16, 25, 36, 49, 64, 81]])
```
Sorting

Sorting is along the “last” axis by default. Note contrast with `np.any()`. To treat the array as a single vector, `axis` must be set to `None`.

Original array is unchanged by use of `np.sort()`, like Python’s built-in `sorted()`.
A cautionary note

**numpy/scipy** have several similarly-named functions with different behaviors!

**Example:** `np.amax, np.ndarray.max, np.maximum`

The best way to avoid these confusions is to
1) Read the documentation carefully
2) Test your code!